

Exercise 8

1. Suppose that X_1, \dots, X_n and Y_1, \dots, Y_n are two independent random samples from two exponential distributions with mean μ_1 and μ_2 respectively. Find the likelihood ratio test for $H_0 : \mu_1 = \mu_2$ against $H_1 : \mu_1 \neq \mu_2$. Specify the asymptotic distribution of the test statistic under H_0 .
2. A survey of the use a particular product was conducted in four areas, and a random sample of 200 potential users was interviewed in each area. In area i , for $i = 1, 2, 3, 4$, x_i of the 200 said that they used the product. Construct a likelihood ratio test to test whether the proportion of the population using the product is the same in each of the four areas. Carry out the test at 5% level for the case $x_1 = 76, x_2 = 53, x_3 = 59$ and $x_4 = 48$.
3. A random sample X_1, \dots, X_n of size n is selected from a normal distribution with known mean μ and unknown variance σ^2 . Two possible confidence intervals for σ^2 are shown below, where a_1, a_2, b_1 and b_2 are constants.

$$(a_1^{-1} \sum_{i=1}^n (X_i - \bar{X})^2, a_2^{-1} \sum_{i=1}^n (X_i - \bar{X})^2), \quad (b_1^{-1} \sum_{i=1}^n (X_i - \mu)^2, b_2^{-1} \sum_{i=1}^n (X_i - \mu)^2).$$

For the case $n = 10$, find values of these constants which give intervals with confidence level 0.90. Compare the expected lengths of these intervals. Comment on your findings.

4. Let X_1, \dots, X_n be a random sample from the uniform distribution on the interval $[0, \theta]$ ($\theta > 0$). Find a confidence interval for θ .