Exercise 6

1. Let $Y_1, \cdots, Y_n$ be a sample from a Poisson distribution with mean $\theta > 0$ unknown.
   (a) Show that the sum $Y \equiv Y_1 + \cdots + Y_n$ is a sufficient statistic. Find its mean and variance. What is the distribution of Y?
   (b) Obtain the MLE for θ.
   (c) Suppose now that only the first m $(m < n)$ observations of the sample are known explicitly, while for the other $n - m$ only their sum, Z say, is known, determine the MLE of θ.
   (d) Compare the answers to (b) and (c), and comment.

2. Find the maximum likelihood estimator of λ given a random sample from the gamma distribution with density function
$$f(x) = \frac{1}{\Gamma(r)} \exp(-\lambda x) x^{r-1} \lambda^r.$$
   Assume that r is known.

3. Find the maximum likelihood estimator for θ from a random sample from the population with density function
$$f(y; \theta) = \frac{2y}{\theta^2} \quad 0 < y \leq \theta, \theta > 0.$$
   Do not use calculus. Draw a picture of the likelihood.

4. Suppose that we have a random sample of size n from the logistic distribution with density function
$$f(y; \mu) = \frac{\exp(y - \mu)}{[1 + \exp(y - \mu)]^2}.$$
   Find the likelihood equation for μ, and write the iteration equations for the Fisher's score method. (Hint: write quantities as much as possible in terms of the density function and the distribution function for the logistic.)

