Summer 2007 Examination

ST305

Actuarial Mathematics: Life
Suitable for all students

Instructions to candidates

Time allowed: 3 hours

This paper contains 6 questions. Answer **All** questions.

You may use an electronic calculator (as prescribed by the examination regulations)

Taxes and expenses should be ignored unless otherwise indicated.

1. Describe in plain English the contracts whose expected present values are given below.

   (a) $\bar{a}_{\overline{5|}} + {}_5p_{65}v^5\bar{a}_{70}$ (3 *marks*)

   (b) $\bar{a}_{\overline{5|}} + v^5\bar{a}_{65}$ (3 *marks*)

   (c) $\int_0^5 v^t {}_tp_x\mu_{x+t}dt + \int_5^\infty v^t {}_{t-5}p_y\, {}_tp_x\mu_{x+t}dt$ (3 *marks*)

   (Total: 9 *marks*)

2. An office issues an n-year unit-linked endowment assurance policy to a life aged x. The policy is financed by a continuous premium payable at rate π. At any time t $(t < n)$ a proportion $(1 - \gamma_t)$ of the premium is invested in a unit fund that is assumed to grow at a constant force r. At time n or on earlier death the policyholder will receive the accumulated amount of the fund or a guaranteed sum g, whichever is larger. A proportion γ_t is allocated to a cash fund and it is calculated in such a way that the net cash flow to or from the cash fund at any time t $(0 < t < n)$ is 0. Show that provided

   $$g\bar{A}_{x:\overline{n|}} \leq \pi\bar{a}_{x:\overline{n|}}$$

   there is no need for the office to set up cash fund reserves. Explain how you would calculate γ_t and the amount the policyholder will receive if he survives to time n. (11 *marks*)

3. A pension plan provides a lump sum benefit at the age of 65 of 5 times the maximum career salary. The member who is entitled to the benefit is now aged 30. The force of interest is assumed to be deterministic and given by $r(t)$. The salary is also calculated as a deterministic function, where the salary of the member at time t is given by $S_0 \exp\left(\int_0^t a(s)\, ds\right)$, where S_0 is the current salary. Note that because of a special salary structure, $a(t) > 0$ for $t < 25$ and $a(t) < 0$ for $t > 25$. Assume a force of mortality μ_{30+t} at time t and expenses at a constant rate c. The plan is financed by a single premium payable upfront.

   (a) Write down an integral expression for the premium. (6 *marks*)

   (b) Suppose now that the office assumes that $r(t)$ follows a stochastic model with two states $r^{(1)}$ and $r^{(2)}$. It is assumed that $r(0) = r^{(1)}$. It is also assumed that there are no expenses and that financial and mortality risks are independent. The transition rates from one state to another are equal and given by λ. The salary structure is deterministic and as already described. Derive differential equations that will help you calculate the expected discounted value of the benefit at any time t and explain how you would use them to calculate the premium. (8 *marks*)

(Total: 14 *marks*)

4. An employee of a company can be working in one of their two sites. The rate of transition from site 1 to site 2 is 0.1 and the rate of transition from site 2 to site 1 is also 0.1. The employee who is aged $x = 35$ is also subject to a force of mortality μ_{x+t} at time t regardless of where he is working. Let $p_1(t)$ denote the probability that he is at site 1 at time t and $p_2(t)$ denote the probability that he is at site 2 at time t. The following quantities are provided (you might not need all of them):

$$_{30}p_{35} = 0.78$$

At a force of interest of 0.05

$\bar{a}_{35:\overline{30|}} = 14.75$ $\qquad$ $(\bar{I}\bar{a})^{\,1}_{35:\overline{30|}} = 1.48$

At a force of interest of 0.15

$\bar{a}_{35:\overline{30|}} = 6.45$ $\qquad$ $(\bar{I}\bar{a})^{\,1}_{35:\overline{30|}} = 0.233$

At a force of interest of 0.25

$\bar{a}_{35:\overline{30|}} = 3.95$ $\qquad$ $(\bar{I}\bar{a})^{\,1}_{35:\overline{30|}} = 0.064$

(a) Write down the forward equations and show that

$$p_1(t) = \left(\frac{1}{2}\exp(-0.2t) + \frac{1}{2}\right) {}_tp_{35}$$

and

$$p_2(t) = \left(\frac{1}{2} - \frac{1}{2}\exp(-0.2t)\right) {}_tp_{35}$$

where

$$_tp_x = \exp\left(-\int_0^t \mu_{x+s}ds\right).$$

(8 *marks*)

(b) The employee is currently at stage 1 and aged 35. The company is offering him a death benefit of 100000 provided death occurs while at state 2. The benefit will stop when the employee reaches the age of 65. Use a force of interest of 0.05 to calculate the expected present value of the benefit. (10 *marks*)

(c) Suppose now that all the benefit in part (b) applies to the first visit to site 2 only and no benefits are applicable for subsequent visits. Calculate the probability that at time t he is in state 2 and this is the first visit as well as the expected present value of the benefit. (8 *marks*)

(Total: 26 *marks*)

5. Consider a health-sickness model with three states "*healthy*", "*sick*" and "*dead*". The sickness rate is $\sigma(t)$, the recovery rate is $\rho(t)$, the mortality rate for a healthy life is $\mu(t)$ and the mortality rate for a sick life is $\nu(t)$. An office is issuing a policy under which a continuous sickness benefit at a rate of b per annum payable while the life is at the "*sick*" state is provided. The policy is financed by a continuous premium payable at a rate π per annum payable while the life is at the "*healthy*' state. The policy duration is 25 years. After that period has elapsed there are no more benefits or premiums.

   (a) Explain by writing down appropriate differential equations and their terminal conditions how the office can calculate π as well as reserves for all possible states at all times $t \leq 25$. (12 *marks*)

   (b) What flaw is there in the design of the policy? (4 *marks*)

   (c) Continuing from (b) explain how you can modify the policy to take care of the problem identified and explain again by writing down appropriate differential equations and their terminal conditions how the office can calculate the premium as well as reserves for all possible states at all times $t \leq 25$. Bear in mind that customers are not willing to pay any lump sums at any time. (10 *marks*)

(Total: 26 *marks*)

6. The next two pages are the printout of a MAPLE worksheet with calculations about a certain life insurance product. The sum of 2805.02 represents the annual premium payable continuously.

   (a) Identify the product by laying out its terms, the benefits, the ages of the life(s) involved, the interest rate and any expenses. (5 *marks*)

   (b) What does the amount of 13732.05 represent? (2 *marks*)

   (c) Ten years after the policy originated the life(s) involved can not pay any more premiums and the policy is converted to a pure endowment with the same maturity date. There will be no more premiums payable, there is a charge for expenses of 300 and there will be a lump sum payable at the end provided all the life(s) involved are still alive. Use the information provided to calculate the lump sum. (7 *marks*)

(Total: 14 *marks*)