Summer 2010 Examination

ST305

Actuarial Mathematics: Life
Suitable for 2010 Candidates

Instructions to candidates

Time allowed: 3 hours

This paper contains 6 questions. Answer **All** questions.

You may use an electronic calculator (as prescribed by the examination regulations)

Taxes and expenses should be ignored unless otherwise indicated.
You may find the following information useful:
If Z is a random variable with a standard normal distribution, $P(|Z| > 1.96) = 0.05$.

1. For a rather old category of lives the 1-year and 2-year survival probabilities are 0.96 and 0.90 respectively. The value of a life annuity relating to this age category payable annually in advance calculated at a force of interest of 0.03 is 7.94. A particular life has passed a medical examination before purchasing the annuity and is therefore subject to a select mortality table, where for the first year the force of mortality is 75 % of the corresponding force for non-select lives and for the second year 90% of the corresponding force for non-select lives. Thereafter there is no difference in mortality. Calculate the value of the life annuity payable annually in advance using a force of interest of 0.03 for this life.

(7 *marks*)

2. The following data were obtained during a one year investigation of the mortality of members of a professional association.

Age group	Number of Deaths	Initial Exposed to Risk
25-35	1	400
36-45	2	1000
46-55	4	1500
56+	18	2100

The following are one year mortality rates for all the members of the profession worldwide.

Age group	q_x
25-35	0.0008
36-45	0.0010
46-55	0.0040
56+	0.0110

(a) Calculate the crude death rate and the standardised mortality ratio for members of the association using the worldwide experience as standard. (3 *marks*)

(b) Is there any statistical evidence that the mortality of association members is different to the mortality of members of the profession worldwide? (3 *marks*)

(Total: 6 *marks*)

3. An office issues an n-year unit-linked endowment assurance policy to a life aged x. The policy is financed by a continuous premium payable at rate π. At any time t $(< n)$ a proportion $(1 - \gamma_t) < 1$ of the premium is invested in a unit fund that is assumed to grow at r_t that might be a stochastic process. A proportion γ_t is allocated to a cash fund that grows at a constant force r^* At time n or on earlier death the policyholder will receive the accumulated amount of the fund or a guaranteed sum b, whichever is larger. γ_t it is calculated in such a way that

$$\gamma_t \pi = (1 + \alpha)\, \mu_{x+t}\, (b - U_t)_+ \,,$$

where U_t is the value of the unit fund at time t, μ_{x+t} is the force of mortality and $\alpha \geq 0$ is a constant.

   (a) Explain why it is not prudent to set $\alpha = 0$. (5 *marks*)

   (b) Explain by deriving expressions for the accumulation of both funds how the office will calculate π if it assumes that $r_t \equiv r > r^*$ for all t. You may assume that with our assumptions $U_n \leq b$. (8 *marks*)

(Total: 13 *marks*)

4. Consider a health-sickness model with three states "*healthy*", "*sick*" and "*dead*". The sickness rate is $\sigma(t)$, the recovery rate is $\rho(t)$, the mortality rate for a healthy life is $\mu(t)$ and the mortality rate for a sick life is $\nu(t)$. The life in the model is healthy at time 0. The force of interest is r.

   (a) Provide an integral expression for the probability that the life is healthy at time t and has never been sick. (2 *marks*)

   (b) Provide an integral expression for the probability that the life is sick at time t and this is the first ever period of sickness. (4 *marks*)

   (c) Provide an integral expression for the present value of a continuous benefit at a rate of 1 payable during the first period of sickness but not before the life has already been sick for a qualifying period q. No benefit will be paid after time $n > q$. (3 *marks*)

(d) Continuing from (c), suppose that at time $t > q_1$ the life has been sick since time $t - q_1$ ($q_1 < q$) and this is the first period of sickness. Provide an integral expression for the prospective reserve for any future benefits. *(make sure you have considered all possible values of t)* (7 *marks*)

(e) Suppose now that if the benefit in (c) is being paid at time n, then the benefit will continue till the life recovers or dies. Provide an integral expression for the present value of the benefit in this case. (6 *marks*)

(Total: 22 *marks*)

5. An office is issuing n-year with profits endowment assurance policies with sum assured b. The continuous premium of π is calculated using a safe first order basis $\left(r^*, \mu^*_{x+t}\right)$. Assume that the actual force of interest is r_t and the actual force of mortality is μ_{x+t}.

   (a) Write down an expression for the rate at which surplus emerges per survivor at time t *(Define all symbols you use)* (3 *marks*)

   (b) The surplus is used to purchase with profits temporary assurance that expires at time n. The surplus generated by this assurance is also used to purchase more temporary assurance and so on. Explain how you can calculate the sum a policyholder will receive if he dies at time t. (13 *marks*)

(Total: 16 *marks*)

6. A 30 year old man has purchased a pension plan that will provide him with a pension of 100000 per annum payable continuously and commencing at time 35 (measured in years). Upon his death, if it happens after time 35, his wife will start receiving a reduced annuity of 80000 per annum also payable continuously, provided she is alive. The annuity of 80000 will continue being paid for 5 years after her death. If he dies before time 35, the 80000 annuity to the wife will commence at time 35 and will continue for 5 years after her death as well. If his wife dies before him the only benefit will be the pension of 100000, which does not continue after his death. There is also the possibility of divorce which can happen at constant force 0.02. If they get divorced before time 10 the wife loses all her entitlements, so the only benefit payable will also be the pension of 100000 (as if she had died). If they get divorced after time 10, she is entitled to everything as if they were married. The continuous premium is payable for as long as he is alive up to time 35. She is aged 33, but for mortality purposes she can be treated as a 30 year old man. The force of interest is 0.04 per annum.

You might find some or all of the following quantities useful (all annuity functions calculated at a force of interest of 0.04).

$$ {}_{35}p_{30} = 0.77 $$

$$ \bar{a}_{30} = 19.93 $$

$$ \bar{a}_{65} = 10.57 $$

$$ \bar{a}_{65:65} = 7.92. $$

(a) Calculate the premium assuming the office incurs expenses of 5% of the premium. (17 *marks*)

(b) Calculate the prospective reserve at time 35 for all possible cases. (6 *marks*)

(c) Provide a formula for the reserve at time $t > 40$ if the wife has died at time 40 and the husband before time 40 (they did not get divorced). (2 *marks*)

(d) Explain how you will calculate prospective reserves at time $t < 35$ *(You may provide integral expressions or a system of differential equations; full credit will be given for any correct answer).* (7 *marks*)

(e) Explain how a negative reserve might arise at some stage. (3 *marks*)

(f) Suggest a premium structure that will address the problem in (e). (1 *mark*)

(Total: 36 *marks*)