Summer 2011 Examination

ST305

Actuarial Mathematics: Life

Suitable for All Candidates

Instructions to candidates

Time allowed: 3 hours

This paper contains 7 questions. Answer **All** questions.

You may use an electronic calculator (as prescribed by the examination regulations)

Taxes and expenses should be ignored unless otherwise indicated.

1. Consider two independent lives aged x and y where $y > x$. The lives are subject to the same mortality force which is increasing with age. Show that the probability that the life aged y will die first is larger than $\frac{1}{2}$.

(3 *marks*)

2. The age specific mortality rates of an area are lower than the rates of the country it belongs to for all ages. Will the Crude Death Rate for the area be lower than the Crude Death Rate of the country? Using the country mortality rates as standard, will the standardized mortality ratio for this area be less than 1? Illustrate your answers with an example.

(5 *marks*)

3. A pension plan provides a pension starting at the age of 65. The pension will be payable continuously and it will be $\frac{1}{80}$ of the average salary over the member's career for every year of service. The member who is entitled to the benefit is now aged 40 and has 15 years of service. The accumulated amount in the fund is B. The member's current salary is S payable continuously and the member's average salary over the past 15 years was S_A. The force of interest is assumed to be deterministic and given by $r(t)$. The salary is also calculated as a deterministic function, where the salary of the member at time t is given by $S_t = S \exp\left(\int_0^t a(s)\,\mathrm{d}s\right)$, where S is the current salary. Assume a force of mortality μ_{40+t} at time t and expenses at a constant rate c per annum before retirement and none thereafter. The plan is financed by a continuous premium that is equal to αS_t. The premium is payable until the time of retirement. Write down integral expressions and use them to calculate α.

(8 *marks*)

4. Consider two identical and independent lives aged x. Provide expressions for the following three quantities. The expressions should be in terms of the survival function ${}_tp_x$, the constant force of interest r and the annuities $\bar{a}_x$ and $\bar{a}_{xx}$. Your expressions in (b), (c) and (d) should involve no integrals.

   (a) The expected value of K where K is a random variable representing the time interval between the first and second deaths.

   (2 *marks*)

   (b) The probability that both lives were alive at time $t-5$ given that they were both dead at time t.

   (3 *marks*)

   (c) The present value of an amount of 1 payable on the second death.

   (3 *marks*)

   (d) The value of an annuity of 1 per annum payable continuously, commencing on the first death and ending 5 years after the second death.

   (6 *marks*)

5. An office issued with profits 30 year endowment assurance policies to 30 year old lives with a sum assured of €100000. The premium is €1980 per annum payable continuously and was calculated using a force of mortality μ_x, where x the life's age, a force of interest of 0.02 per annum and no expenses. The real force of interest is stochastic and can take the values 0.01 and 0.04. The transition rates between the two values are 0.5. The actual force of mortality is also μ_x at age x. The real force of interest is 0.04 initially. The policyholders will receive all surplus in the form of a cash bonus. The emerging surplus is paid out to policyholders if it is positive and nothing is paid if it is negative. Explain how you would calculate the expected present value of the loss the office will incur over the period of the policy. *(Note that although you are not required to derive any of the equations, some sort of derivation will give you partial credit if things go wrong. You should in any case define the functions involved in the equations).*

   (14 *marks*)

6. Consider a health-sickness Markov model. The sickness rate for people that have never been sick before is σ_x, the sickness rate for people that have been sick before at least once is $\tilde{\sigma}_x$ and the recovery rate from any period of sickness is ρ_x. The mortality rate for a healthy life that has never been sick before is μ_x, the mortality rate for a healthy life that has been sick before is $\tilde{\mu}_x$ and the mortality rate for a sick life is ν_x. An office is issuing a policy under which a continuous sickness benefit at a rate of b per annum payable while the life is sick is provided. The life is now 35 years old. The policy is financed by a continuous premium at a rate π per annum payable while the life is healthy. The policy duration is 25 years. After that period has elapsed there are no more benefits or premiums.

   (a) Explain by writing down appropriate differential equations and their terminal conditions how the office can calculate π.

   (10 *marks*)

   (b) Explain how the company will calculate reserves at all times and states.

   (6 *marks*)

   (c) Suppose now that the policy expires at time 25 if the policyholder is healthy but if she is sick, she continues receiving the benefit for as long as this period of sickness lasts. Upon recovery the policy expires. Explain how the office can calculate π in this case.

   (8 *marks*)

   (d) What problem does the modification in (c) address from the point of view of the office?

   (2 *marks*)

7. The MAPLE output at the end of this question is supposed to be used as a substitute for actuarial tables. It consists of some calculations you will find essential and it is not intended to be the solution to any specific problem.

(a) What do 14.7089 (equation (11) on the worksheet) and 8.4811 (equation (13) on the worksheet) represent?

(3 *marks*)

(b) Calculate the value of a deferred annuity for a 45 year old assuming a deterministic force of interest of 0.04 for the first 5 years and 0.03 thereafter. The annuity commences in 5 years time if the life is alive then and last for 10 years.

(3 *marks*)

(c) A life office is offering 20 year **pure endowment** policies to lives aged 40. The sum of 100000 is payable at the end of the policy if the policyholder survives. There is no death benefit. The policies are financed by a continuous premium payable throughout the term of the policy. The policies might lapse at any time with a force 0.02 (the policyholder decides not to pay any more premiums). Should that happen during the first 5 years the company imposes a charge of 50% of the reserve and uses the remainder as a single premium for any life product the policyholder chooses. If the policy lapses during the last 15 years the charge is not applicable, so the whole reserve is used as a single premium for this product. Calculate the continuous premium.

(13 *marks*)

(d) A particular policy lapses at time 10. The reserve is then used as a single premium to purchase an endowment assurance. The sum assured b is paid at time 20 or on earlier death. Calculate b.

(7 *marks*)

(e) Describe two problems with the assumptions about lapse rates in (c).

(4 *marks*)

This is the definition of the force of mortality for the G82M table.

```
>  m := t -> 0.0005+0.00007585775*10^(0.038*t) ;
```

$$m := t \to 0.0005 + 0.00007585775\ 10^{(0.038\,t)} \quad (1)$$

This is the survival function for a life aged 40

```
> evalf(0.00007585775*10^(0.038*40)/(0.038*ln(10)));
```

$$0.02870785022 \quad (2)$$

```
> p := t -> exp(-0.0005*t-0.02870785022*(10^(0.038*t)-1));
```

$$p := t \to \mathrm{e}^{\left(-0.0005\,t - 0.02870785022\left(10^{(0.038\,t)} - 1\right)\right)} \quad (3)$$

```
> evalf(p(5));
```

$$0.9819102742 \quad (4)$$

```
> evalf(p(10));
```

$$0.9558469375 \quad (5)$$

```
> evalf(p(15));
```

$$0.9180972778 \quad (6)$$

```
> evalf(p(20));
```

$$0.8637355917 \quad (7)$$

```
> deq1  := diff(v(t),t) -  (0.03+m(40+t))*v(t) + 1= 0;
```

$$deq1 := \frac{\mathrm{d}}{\mathrm{d}t}\,v(t) - \left(0.0305 + 0.00007585775\ 10^{(1.140 + 0.038\,t)}\right) v(t) + 1 = 0 \quad (8)$$

```
> tc1   := v(20) = 0;
```

$$tc1 := v(20) = 0 \quad (9)$$

```
> dsol1 := dsolve({deq1,tc1}, numeric, range=0..20);
```

$dsol1$:= **proc**(x_rkf45) ...**end proc** (10)

```
> dsol1(0);
```

$$[t = 0.,\ v(t) = 14.7088614232782682] \quad (11)$$

```
> dsol1(5);
```

$$[t = 5.,\ v(t) = 11.8240148160107790] \quad (12)$$

```
> dsol1(10);
```

$$[t = 10.,\ v(t) = 8.48107956713589318] \quad (13)$$

```
> dsol1(15);
```

$$[t = 15.,\ v(t) = 4.58676899946821858] \quad (14)$$

```
> dsol1(20);
```

$$[t = 20.,\ v(t) = 1.] \quad (15)$$

```
> deq2  := diff(v(t),t) -  (0.04+m(40+t))*v(t) + 1= 0;
```

$$deq2 := \frac{\mathrm{d}}{\mathrm{d}t}\,v(t) - \left(0.0405 + 0.00007585775\ 10^{(1.140 + 0.038\,t)}\right) v(t) + 1 = 0 \quad (16)$$

```
> tc2   := v(20) = 0;
```

(17)

$$tc2 := v(20) = 0 \tag{17}$$

```
> dsol2 := dsolve({deq2,tc2}, numeric, range=0..20);
```

$$dsol2 := \mathbf{proc}(x\_rkf45)\ \ldots\mathbf{end\ proc} \tag{18}$$

```
> dsol2(0);
```

$$[t = 0., v(t) = 13.4775390710781268] \tag{19}$$

```
> dsol2(5);
```

$$[t = 5., v(t) = 11.0490489083441190] \tag{20}$$

```
> dsol2(10);
```

$$[t = 10., v(t) = 8.09392579345781016] \tag{21}$$

```
> dsol2(15);
```

$$[t = 15., v(t) = 4.47728109678012310] \tag{22}$$

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>
>
>
```