

Solutions to Extra Exercise 0

1. Probability he will die within 10 years,

$$\begin{aligned} {}_{10}q_{60} &= 1 - {}_{10}p_{60} \\ &= 1 - 0.7897 \\ &= 0.2103 \end{aligned}$$

2. His expected remaining lifetime is

$$\begin{aligned} E(T_{60}) &= \int_0^\infty {}_tp_{60}\mu_{60+t}dt \\ &= \int_0^\infty {}_tp_{60}dt \\ &= 18.5239 \end{aligned}$$

And the variance is

$$\begin{aligned} E(T^2) &= \int_0^\infty t^2 {}_tp_{60}\mu_{60+t}dt \\ &= \int_0^\infty 2t {}_tp_{60}dt \quad (\text{integration by parts}) \\ &= 430.4206 \\ \text{Var}(T) &= 430.426 - (18.5239)^2 \\ &= 87.286 \end{aligned}$$

3. The required probability is

$$\begin{aligned} P(5 < T_{60} < 10 | T_{60} < 10) &= \frac{P(5 < T_{60} < 10)}{P(T_{60} < 10)} \\ &= \frac{{}_{10}q_{60} - {}_5q_{60}}{{}_{10}q_{60}} \\ &= \frac{{}_5p_{60} - {}_{10}p_{60}}{1 - {}_{10}p_{60}} \\ &= \frac{0.911 - 0.7897}{0.2103} \\ &= 0.5768 \end{aligned}$$

4. His expected remaining lifetime at age 60 given that he died within 10 years,

$$\begin{aligned} E(K) &= \int_0^{10} t \frac{{}_tp_{60}\mu_{60+t}}{0.2103} dt \\ &= \frac{1.159}{0.2103} \\ &= 5.511 \end{aligned}$$