Solutions to Extra Exercise 17

1.
$$p_1(t) = e^{-0.1t}{}_tp_x$$

$$\begin{aligned}
p_2(t) &= \int_0^t p_1(s)0.1p_{22}(s,t)ds \text{ } (p_{22} = p_{\overline{22}}, \text{ so does not matter which one is used}) \\
&= 0.1\int_0^t e^{-0.1s}{}_sp_xe^{-0.2(t-s)}{}_{t-s}p_{x+s}ds \\
&= 0.1{}_tp_x\int_0^t e^{-0.2t}e^{0.1s}ds \\
&= {}_tp_x(e^{-0.1t} - e^{-0.2t}) \\
p_3(t) &= \int_0^t p_2(s)0.2p_{\overline{33}}(s,t)ds \\
&= \int_0^t {}_sp_x(e^{-0.15} - e^{-0.25})0.2e^{-0.6(t-s)}{}_{t-s}p_{x+s}ds \\
&= {}_tp_x\int_0^t 0.2e^{-0.6t}(e^{0.5s} - e^{0.4s})ds \\
&= {}_tp_x0.2e^{-0.6t}\left(\frac{e^{0.5t}-1}{0.5} - \frac{e^{0.4t}-1}{0.4}\right) \\
&= {}_tp_x(0.4e^{-0.1t} - 0.5e^{-0.2t} + 0.1e^{-0.6t})
\end{aligned}$$

2. The required probability is the probability that the life dies while at state 1 or dies while at state 2.

$$\begin{aligned}
& P(\text{dies while at state 1}) + P(\text{dies while at state 2}) \\
=& \int_0^\infty p_1(t)\mu_{x+t}dt + \int_0^\infty p_2(t)\mu_{x+t}dt \\
=& \int_0^\infty e^{-0.1t}{}_tp_x\mu_{x+t}dt + \int_0^\infty (e^{-0.1t} - e^{-0.2t}){}_tp_x\mu_{x+t}dt \\
=& \bar{A}_{20}^{(0.1)} + \bar{A}_{20}^{(0.1)} - \bar{A}_{20}^{(0.2)} \\
=& 2\bar{A}_{20}^{(0.1)} - \bar{A}_{20}^{(0.2)} \\
=& 2(1 - 0.1\times 9.284) - (1 - 0.2\times 4.480) \\
=& 0.0392
\end{aligned}$$

3. Start by calculating the case where we have life cover no matter what

$$
\begin{aligned}
C_1 &= 50000\left(\int_0^\infty e^{-0.05t}p_2(t)\mu_{x+t}dt + \int_0^\infty e^{-0.05t}p_3(t)(\mu_{x+t}+0.6)dt + \int_0^\infty e^{-0.05t}p_3(t)dt\right)\\
&= 50000\left(\int_0^\infty e^{-0.05t}(e^{-0.1t}-e^{-0.2t})_tp_x\mu_{x+t}dt\right.\\
&\quad + \int_0^\infty e^{-0.05t}(0.4e^{-0.1t}-0.5e^{-0.2t}+0.1e^{-0.6t})_tp_x(\mu_{x+t}+0.6)dt\\
&\quad \left.+ \int_0^\infty e^{-0.05t}(0.4e^{-0.1t}-0.5e^{-0.2t}+0.1e^{-0.6t})_tp_x dt\right)\\
&= 50000\left(\bar{A}_{20}^{(0.15)} - \bar{A}_{20}^{(0.25)} + 0.4\bar{A}_{20}^{(0.15)} - 0.5\bar{A}_{20}^{(0.25)} + 0.1\bar{A}_{20}^{(0.65)}\right.\\
&\quad \left.+1.6(0.4\bar{a}_{20}^{(0.15)} - 0.5\bar{a}_{20}^{(0.25)} + 0.1\bar{a}_{20}^{(0.65)}\right)
\end{aligned}
$$

Now, we subtract the part after 40 years.

$$V_1(40) = \bar{A}_{60}^{(0.15)} - \bar{A}_{60}^{(0.15)} - 0.5\bar{A}_{60}^{(0.25)} + 0.1\bar{A}_{60}^{(0.65)} + 1.6(0.4\bar{a}_{60}^{(0.15)} - 0.5\bar{a}_{60}^{(0.25)} + 0.1\bar{a}_{60}^{(0.65)}$$

The EPV of benefits is thus $C_1 - 0.845e^{-0.05\times 40}V_1(40)$.