Solutions to Extra Exercise 3

1 invested at time t_1 accumulates to $\frac{e^{(0.07-0.04e^{-0.03t_2})t_2}}{e^{(0.07-0.04e^{-0.03t_1})t_1}}$ at t_2.

1 paid at t_2 is worth the reciprocal at t_1.

At time 0, $e^{-(0.07-0.04e^{-0.03t_2})t_2}$.

$$\begin{aligned}
\text{EPV(benefits)} &= 30000\int_{29.5}^{\infty} {}_tp_{35.5}e^{-(0.07-0.04e^{-0.03t})t}dt + 100000\int_0^{29.5} {}_tp_{35.5}\mu_{35.5+t}e^{-(0.07-0.04e^{-0.03t})t}dt \\
&= 30000\times 1.3523 + 100000\times 0.09322 \\
&= 49891 \\
\text{EPV(premiums)} &= \pi\int_0^{29.5} {}_tp_{35.5}e^{-(0.07-0.04e^{-0.03t})t}dt \\
&= 15.4549\pi
\end{aligned}$$

So

$$\begin{aligned}
\pi &= \frac{49891}{15.4549} \\
&= 3228
\end{aligned}$$

<u>2nd Oct 2021</u>

(i) Prospective:

$$\begin{aligned}
&30000\int_{14.5}^{\infty} {}_tp_{50.5}e^{-(0.07-0.04e^{-0.03t})t}dt \\
&+100000\int_0^{14.5} {}_tp_{50.5}\mu_{50.5+t}e^{-(0.07-0.04e^{-0.03t})t}dt \\
&-3228\int_0^{14.5} {}_tp_{50.5}e^{-(0.07-0.04e^{-0.03t})t}dt \\
=\ &30000\times 3.9233 + 100000\times 0.12552 - 3228\times 10.3440 \\
=\ &96861
\end{aligned}$$

(ii) Retrospective:

$$\begin{aligned}
&3228\int_0^{15} \frac{e^{-(0.07-0.04e^{-0.03t})t}}{e^{-(0.07-0.04e^{-0.03\times 15})15}}\frac{1}{{}_{15-t}p_{35.5+t}}dt \\
&-100000\int_0^{15} \mu_{35.5+t}\frac{e^{-(0.07-0.04e^{-0.03t})t}}{e^{-(0.07-0.04e^{-0.03\times 15})15}}\frac{1}{{}_{15-t}p_{35.5+t}}dt \\
=\ &3228\times 22.9102 - 100000\times 0.0857 \\
=\ &65384
\end{aligned}$$