Solutions to Extra Exercise 5

1. We are given:

$$\begin{aligned} {}_1p_x &= 0.94 \\ {}_2p_x &= 0.87 \\ \ddot{a}_x &= 7.52 \end{aligned}$$

So ${}_1p_{x+1} = 0.87/0.94 = 0.9255$. We can calculate the survival probability for the first two years for the selected individual.

$$\begin{aligned} {}_1q_x = 0.06 \Rightarrow {}_1q_{[x]} = 0.048 \quad &\Rightarrow \quad {}_1p_{[x]} = 0.952 \\ {}_1q_{x+1} = 0.0745 \Rightarrow {}_1q_{[x]+1} = 0.06705 \quad &\Rightarrow \quad {}_1p_{[x]+1} = 0.93295 \end{aligned}$$

Now, we want to calculate $\ddot{a}_{[x]}$. We have

$$\begin{aligned} \ddot{a}_x &= 1 + e^{-0.04}{}_1p_x + e^{-0.08}{}_2p_x + \sum_{t=3}^{\infty} e^{-0.04t}{}_tp_x \\ &= 1 + e^{-0.04} \times 0.94 + e^{-0.08} \times 0.87 + {}_2p_x \sum_{t=3}^{\infty} e^{-0.04t}{}_{t-2}p_{x+2} \\ &= 7.52 \end{aligned}$$

So

$$\sum_{t=3}^{\infty} e^{-0.04t}{}_{t-2}p_{x+2} = 5.533$$

Now we can get $\ddot{a}_{[x]}$.

$$\begin{aligned} \ddot{a}_{[x]} &= 1 + e^{-0.04}{}_1p_{[x]} + e^{-0.08}{}_2p_{[x]} + \sum_{t=3}^{\infty} e^{-0.04t}{}_tp_{[x]} \\ &= 1 + e^{-0.04}{}_1p_{[x]} + e^{-0.08}\left({}_1p_{[x]} \times {}_1p_{[x]+1}\right) + \sum_{t=3}^{\infty} e^{-0.04t}\left({}_1p_{[x]} \times {}_1p_{[x]+1}\right) \times {}_{t-2}p_{x+2} \\ &= 7.649 \end{aligned}$$

2. Now the force of mortality changes.

$$\begin{aligned} {}_1p_{[x]} &= e^{-\int_0^1 \mu_{[x]+s}ds} \\ &= e^{-\int_0^1 0.8\times\mu_{x+s}ds} \\ &= \left({}_1p_x\right)^{0.8} \\ &= 0.9517 \\ {}_1p_{[x]+1} &= e^{-\int_1^2 \mu_{[x]+s}ds} \\ &= \left({}_1p_{x+1}\right)^{0.9} \\ &= 0.9327 \end{aligned}$$

Similar to above, we get

$$
\begin{aligned}
\ddot{a}_{[x]} &= 1 + e^{-0.04}\,{}_{1}p_{[x]} + e^{-0.08}\,{}_{2}p_{[x]} + \sum_{t=3}^{\infty} e^{-0.04t}\,{}_{t}p_{[x]} \\
&= 1 + e^{-0.04}\,{}_{1}p_{[x]} + e^{-0.08}\left({}_{1}p_{[x]} \times {}_{1}p_{[x]+1}\right) + \sum_{t=3}^{\infty} e^{-0.04t}\left({}_{1}p_{[x]} \times {}_{1}p_{[x]+1}\right) \times {}_{t-2}p_{x+2} \\
&= 7.645
\end{aligned}
$$