Solutions to Extra Exercise 6

1. Single premium

$$\begin{aligned}\text{EPV(benefits)} &= \int_0^{20} e^{-rt}\left({}_tp_{40\,t}p_{45}\mu_{45+t\,t}q_{50} + {}_tp_{40\,t}p_{50}\mu_{50+t\,t}q_{45}\right.\\ &\quad + {}_tp_{45\,t}p_{40}\mu_{40+t\,t}q_{50} + {}_tp_{45\,t}p_{50}\mu_{50+t\,t}q_{40}\\ &\quad \left.+ {}_tp_{50\,t}p_{45}\mu_{45+t\,t}q_{40} + {}_tp_{50\,t}p_{40}\mu_{40+t\,t}q_{45}\right) dt\\ &= 0.0593\end{aligned}$$

   Since EPV(premiums) = EPV(benefits) + EPV(expenses),

$$\begin{aligned}0.95\pi &= 59300\\ \pi &= 62421\end{aligned}$$

2. Continuous premium payable as long as all partners are alive and up to 10 years

$$\begin{aligned}\bar{a}_{40:45:50:\overline{10}|} &= \int_0^{10} e^{-rt}{}_tp_{40\,t}p_{45\,t}p_{50}dt\\ &= 7.5799\end{aligned}$$

   So

$$\begin{aligned}0.95\pi(7.5799) &= 59300\\ \pi &= 8235\end{aligned}$$

3. Continuous premium payable as long as two lives are alive up to 10 years

$$\begin{aligned}\text{Value of annuity} &= \int_0^{10} e^{-rt}\left({}_tp_{40\,t}p_{45\,t}p_{50} + {}_tp_{40\,t}p_{45\,t}q_{50} + {}_tp_{40\,t}q_{45\,t}p_{50} + {}_tq_{40\,t}p_{45\,t}p_{50}\right) dt\\ &= 7.5799 + 0.6358 = 8.2157\end{aligned}$$

   So

$$\begin{aligned}0.95\pi(8.2157) &= 59300\\ \pi &= 7598\end{aligned}$$

Suppose now we know that the sum assured was paid. To find the probability that A was the survivor:

$$\begin{aligned}P(\text{A was survivor}|\text{2 deaths before 20}) &= \frac{P(\text{A last to die, other 2 die before t=20})}{P(\text{2 deaths before 20})}\\ &= \frac{\int_0^{20}\left({}_tp_{40\,t}p_{45}\mu_{45+t\,t}q_{50} + {}_tp_{40\,t}p_{50}\mu_{50+t\,t}q_{45}\right) dt}{\int_0^{20}\left(\text{6 terms as in first part}\right) dt}\\ &= \frac{0.05177}{0.1075}\\ &= 0.4816\end{aligned}$$

Suppose that the sum was paid 15 years exactly after the policy took effect. The probability that A was the survivor:

$$P(\text{A got the money}|\text{sum was paid at year 15}) = \frac{{}_{15}p_{40}\,{}_{15}p_{45}\,\mu_{60}\,{}_{15}q_{50} + {}_{15}p_{40}\,{}_{15}q_{45}\,{}_{15}p_{50}\,\mu_{65}}{P(\text{sum was paid at year 15})}$$

$$\begin{aligned} P(\text{sum was paid at year 15}) &= {}_{15}p_{40}\,{}_{15}p_{45}\,\mu_{60}\,{}_{15}q_{50} + {}_{15}p_{40}\,{}_{15}p_{50}\,\mu_{65}\,{}_{15}q_{45} \\ &\quad + {}_{15}p_{45}\,{}_{15}p_{40}\,\mu_{55}\,{}_{15}q_{50} + {}_{15}p_{45}\,{}_{15}p_{50}\,\mu_{65}\,{}_{15}q_{40} \\ &\quad + {}_{15}p_{50}\,{}_{15}p_{45}\,\mu_{60}\,{}_{15}q_{40} + {}_{15}p_{50}\,{}_{15}p_{40}\,\mu_{55}\,{}_{15}q_{45} \\ &= 0.008759 \end{aligned}$$

So the required probability is $\frac{0.004217}{0.008759} = 0.4814$.