

## Solutions to Extra Exercise 7

1. Condition on life 1 dying at time  $t$ , the probability that life 2 dies within  $k$  years is  ${}_t-kp_x - {}_{t+k}p_x$ .

$$\begin{aligned}
 \int_0^\infty {}_tp_x \mu_{x+t} ({}_t-kp_x - {}_{t+k}p_x) dt &= \int_0^k {}_tp_x \mu_{x+t} (1 - {}_{t+k}p_x) dt + \int_k^\infty {}_tp_x \mu_{x+t} ({}_t-kp_x - {}_{t+k}p_x) dt \\
 &= \int_0^k {}_tp_x \mu_{x+t} dt + \int_k^\infty {}_tp_x \mu_{x+t} {}_t-kp_x dt - \int_0^\infty {}_tp_x \mu_{x+t} {}_{t+k}p_x dt \\
 &= \int_0^k {}_tp_x \mu_{x+t} dt + \int_0^\infty {}_{s+k}p_x \mu_{x+s+k} {}_sp_x ds - \int_0^\infty {}_tp_x \mu_{x+t} {}_{t+k}p_x dt \\
 &= \int_0^k {}_tp_x \mu_{x+t} dt + \int_0^\infty {}_{t+k}p_x {}_tp_x (\mu_{x+t+k} - \mu_{x+t}) dt \\
 &> \int_0^k {}_tp_x \mu_{x+t} dt \quad (\text{since mortality increases with time}) \\
 &= {}_kq_x
 \end{aligned}$$

2. Let  $T$  be the interval and  $T_1$  the time of death of life 1.

$$\begin{aligned}
 &P(T \leq k) > P(T_1 \leq k) \quad (\text{from part 1.}) \\
 \Rightarrow &P(T > k) < P(T_1 > k) \\
 \Rightarrow &\int_0^\infty P(T > k) dk < \int_0^\infty P(T_1 > k) dk \\
 \Rightarrow &E(T) < E(T_1)
 \end{aligned}$$