

Solutions to Extra Exercise 8

1. Condition on life 40 dying at time t . Then the probability the 45 year old dies (or died) within 10 years is ${}_t-10p_{45} - {}_t+10p_{45}$. Required probability is

$$\begin{aligned}
 & \int_0^\infty {}_tp_{40}\mu_{40+t}({}_t-10p_{45} - {}_t+10p_{45})dt \\
 = & \int_0^{10} {}_tp_{40}\mu_{40+t}(1 - {}_t+10p_{45})dt + \int_{10}^\infty {}_tp_{40}\mu_{40+t}({}_t-10p_{45} - {}_t+10p_{45})dt \\
 = & 0.00584 + 0.41323 \\
 = & 0.419
 \end{aligned}$$

2. The time interval between their two deaths is the time interval when there is exactly one life alive. So the expected value is

$$\begin{aligned}
 E\left(\int_0^\infty \mathbf{1}_{\{\text{exactly one life alive at time } t\}}dt\right) &= \int_0^\infty P(\text{exactly one life alive at } t)dt \\
 &= \int_0^\infty ({}_tp_{45}q_{40} + {}_tp_{40}q_{45})dt \\
 &= 14.418
 \end{aligned}$$

3. Similarly, the time interval between the first and last death is the time interval when there are either one or two lives alive. The expected value is

$$\begin{aligned}
 E\left(\int_0^\infty \mathbf{1}_{\{\text{exactly two lives alive at time } t\}}dt\right) &= \int_0^\infty P(\text{exactly two lives alive at time } t)dt \\
 &= \int_0^\infty (1 - {}_tp_{40}{}_tp_{45}{}_tp_{50} - {}_tq_{40}{}_tq_{45}{}_tq_{50})dt \\
 &= 21.747
 \end{aligned}$$