```
> m := t -> 0.0005+0.00007585775*10^(0.038*t);
```

$$m := t \to 0.0005 + 0.00007585775\ 10^{0.038\,t} \tag{1}$$

```
> m2 := t -> 0.3/(100-t);
```

$$m2 := t \to \frac{0.3}{100 - t} \tag{2}$$

```
> This is to caclulate the present value of the benefit
> dsys1 := {diff(w0(t),t)=0.04*w0(t)-m(30+t)*(w2(t)-w0(t))-m(35+t)
  *(w1(t)-w0(t))-m2(t)*(100000-w0(t)), diff(w1(t),t)=0.04*w1(t)-(m
  (30+t)+m2(t))*(100000-w1(t)),  diff(w2(t),t)=0.04*w2(t)-(m(35+t)
  +m2(t))*(100000-w2(t)), w0(30)=0, w1(30)=0, w2(30)=0};
>
```

$$dsys1 := \left\{w0(30) = 0,\ w1(30) = 0,\ w2(30) = 0,\ \frac{\mathrm{d}}{\mathrm{d}t}\ w0(t) = 0.04\ w0(t) - (0.0005 + 0.00007585775\ 10^{1.140 + 0.038\,t})\ (w2(t) - w0(t)) - (0.0005 + 0.00007585775\ 10^{1.330 + 0.038\,t})\ (w1(t) - w0(t)) - \frac{0.3\ (100000 - w0(t))}{100 - t},\ \frac{\mathrm{d}}{\mathrm{d}t}\ w1(t) = 0.04\ w1(t) - \left(0.0005 + 0.00007585775\ 10^{1.140 + 0.038\,t} + \frac{0.3}{100 - t}\right)\ (100000 - w1(t)),\ \frac{\mathrm{d}}{\mathrm{d}t}\ w2(t) = 0.04\ w2(t) - \left(0.0005 + 0.00007585775\ 10^{1.330 + 0.038\,t} + \frac{0.3}{100 - t}\right)\ (100000 - w2(t))\right\} \tag{3}$$

```
> dsol1 := dsolve(dsys1, numeric, range = 0..30);
```

$$dsol1 := \mathbf{proc}(x\_rkf45)\ \ldots\ \mathbf{end\ proc} \tag{4}$$

```
> dsol1(30);
```

$$[t = 30.,\ w0(t) = 0.,\ w1(t) = 0.,\ w2(t) = 0.] \tag{5}$$

```
> dsol1(20);
```

$$[t = 20.,\ w0(t) = 4222.26267580244166,\ w1(t) = 10691.9754326356924,\ w2(t) = 14310.8367945067767] \tag{6}$$

```
> dsol1(0);
```

$$[t = 0.,\ w0(t) = 6971.28621628026304,\ w1(t) = 12441.2552546313309,\ w2(t) = 15370.5091565596104] \tag{7}$$

```
> We will now calculate the valur of a 30 year annuity payable as long as the two lives are alive.
> deq1   := diff(u(t),t) =(0.04+m(35+t)+m(30+t)+m2(t))*u(t) -1 ;
```

$$deq1 := \frac{\mathrm{d}}{\mathrm{d}t}\ u(t) = \left(0.0410 + 0.00007585775\ 10^{1.330 + 0.038\,t} + 0.00007585775\ 10^{1.140 + 0.038\,t} + \frac{0.3}{100 - t}\right) u(t) - 1 \tag{8}$$

```
> tc1    := u(30) = 0;
```

$$tc1 := u(30) = 0 \tag{9}$$

```
> dsol2 := dsolve({deq1,tc1}, numeric, range=0..30);
```

$$dsol2 := \mathbf{proc}(x\_rkf45) \ \ldots \ \mathbf{end\ proc} \tag{10}$$

```
> dsol2(30);
```

$$[t = 30., u(t) = 0.] \tag{11}$$

```
> dsol2(20);
```

$$[t = 20., u(t) = 7.33535881107048126] \tag{12}$$

```
> dsol2(0);
```

$$[t = 0., u(t) = 15.3469224922779670] \tag{13}$$

```
> So the premium is:
> evalf(6971.28621628026304/15.3469224922779670);
```

$$454.2465254 \tag{14}$$

```
> dsys2 := {diff(v0(t),t)=0.04*v0(t)-m(30+t)*(v2(t)-v0(t))-m(35+t)
  *(v1(t)-v0(t))-m2(t)*(100000-v0(t))+454.2465254, diff(v1(t),t)=
  0.04*v1(t)-(m(30+t)+m2(t))*(100000-v1(t)),  diff(v2(t),t)=0.04*
  v2(t)-(m(35+t)+m2(t))*(100000-v2(t)), v0(30)=0, v1(30)=0, v2(30)
  =0};
```

$$\begin{aligned} dsys2 := \Big\{ & v0(30) = 0,\ v1(30) = 0,\ v2(30) = 0,\ \frac{\mathrm{d}}{\mathrm{d}t}\, v0(t) = 0.04\, v0(t) - \big(0.0005 \\ & + 0.00007585775\ 10^{1.140 + 0.038\, t}\big)\ (v2(t) - v0(t)) - \big(0.0005 \\ & + 0.00007585775\ 10^{1.330 + 0.038\, t}\big)\ (v1(t) - v0(t)) - \frac{0.3\ (100000 - v0(t))}{100 - t} \\ & + 454.2465254,\ \frac{\mathrm{d}}{\mathrm{d}t}\, v1(t) = 0.04\, v1(t) - \left(0.0005 + 0.00007585775\ 10^{1.140 + 0.038\, t} \right. \\ & \left. + \frac{0.3}{100 - t}\right)\ (100000 - v1(t)),\ \frac{\mathrm{d}}{\mathrm{d}t}\, v2(t) = 0.04\, v2(t) - \left(0.0005 \right. \\ & \left. + 0.00007585775\ 10^{1.330 + 0.038\, t} + \frac{0.3}{100 - t}\right)\ (100000 - v2(t)) \Big\} \end{aligned} \tag{15}$$

```
> dsol3 := dsolve(dsys2, numeric, range = 0..30);
```

$$dsol3 := \mathbf{proc}(x\_rkf45) \ \ldots \ \mathbf{end\ proc} \tag{16}$$

```
> dsol3(30);
```

$$[t = 30., v0(t) = 0., v1(t) = 0., v2(t) = 0.] \tag{17}$$

```
> dsol3(25);
```

$$[t = 25., v0(t) = 400.237897471181213, v1(t) = 7083.92505017044642, v2(t) = 9713.87706454217550] \tag{18}$$

```
> dsol3(20);
```

$$[t = 20., v0(t) = 890.202872662893014, v1(t) = 10691.9740882724874, v2(t) \tag{19}$$

$= 14310.8343051310640]$

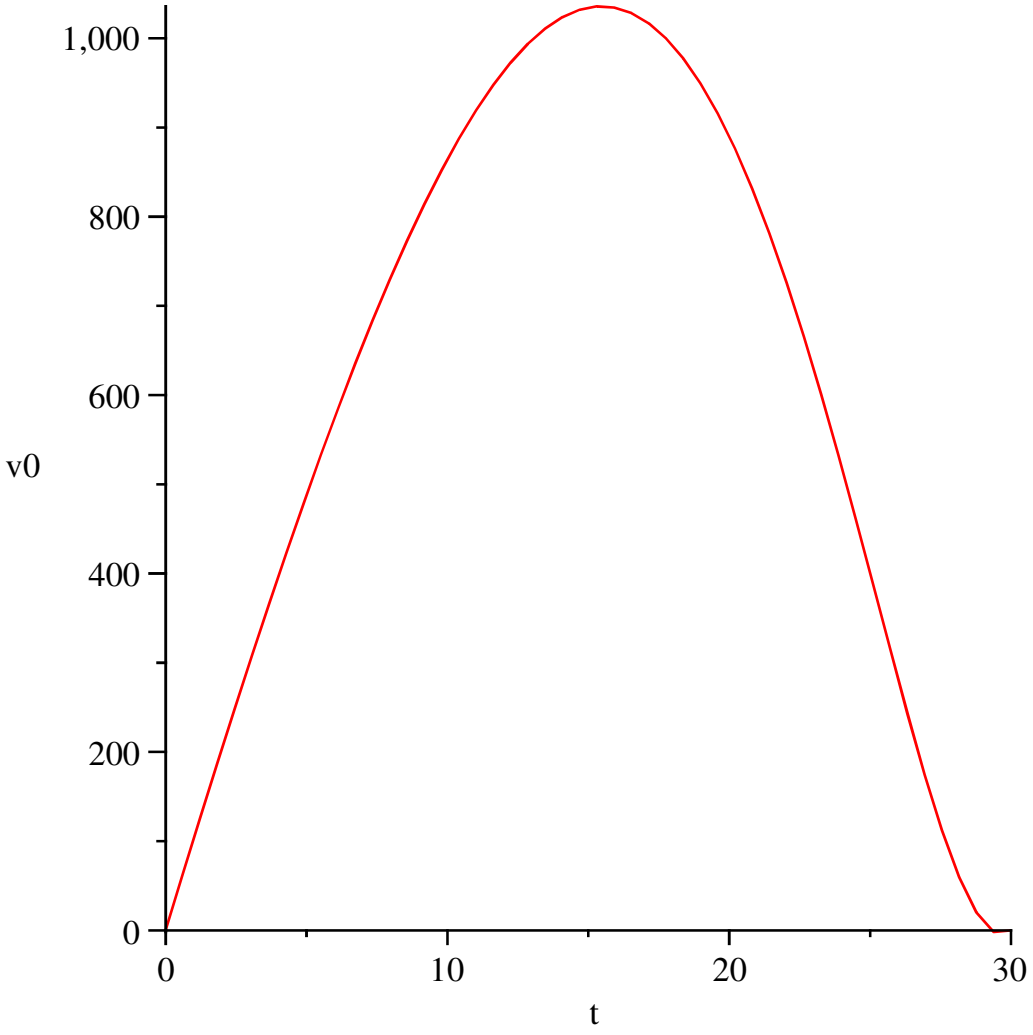
```
> dsol3(0);
```

$[t = 0., v0(t) = -0.00005864077570549852 68, v1(t) = 12441.2546284530909, v2(t)$ **(20)**
$= 15370.5083317145473]$

```
> with(plots);
```

[*animate*, *animate3d*, *animatecurve*, *arrow*, *changecoords*, *complexplot*, *complexplot3d*, **(21)**
conformal, *conformal3d*, *contourplot*, *contourplot3d*, *coordplot*, *coordplot3d*, *densityplot*,
display, *fieldplot*, *fieldplot3d*, *gradplot*, *gradplot3d*, *graphplot3d*, *implicitplot*,
implicitplot3d, *inequal*, *interactive*, *interactiveparams*, *intersectplot*, *listcontplot*,
listcontplot3d, *listdensityplot*, *listplot*, *listplot3d*, *loglogplot*, *logplot*, *matrixplot*, *multiple*,
odeplot, *pareto*, *plotcompare*, *pointplot*, *pointplot3d*, *polarplot*, *polygonplot*, *polygonplot3d*,
polyhedra_supported, *polyhedraplot*, *rootlocus*, *semilogplot*, *setcolors*, *setoptions*,
setoptions3d, *spacecurve*, *sparsematrixplot*, *surfdata*, *textplot*, *textplot3d*, *tubeplot*]

```
> odeplot(dsol3);
```




```
> dsol3(29.8);
```

(22)

$$[t = 29.8,\ v0(t) = -3.56446735088006506,\ v1(t) = 379.911695044891928,\ v2(t) = 535.839344813989442] \quad (22)$$

```
> dsol3(29.7);
```

$$[t = 29.7,\ v0(t) = -4.21562582442145572,\ v1(t) = 566.245259569357132,\ v2(t) = 798.124217055733994] \quad (23)$$

```
> dsol3(29.6);
```

$$[t = 29.6,\ v0(t) = -4.15312676997262464,\ v1(t) = 750.203930943616115,\ v2(t) = 1056.72211505050404] \quad (24)$$

```
> dsol3(29.5);
```

$$[t = 29.5,\ v0(t) = -3.40614254083939416,\ v1(t) = 931.817153149278965,\ v2(t) = 1311.68415986150785] \quad (25)$$

```
> dsol3(29.4);
```

$$[t = 29.4,\ v0(t) = -2.00297015868153583,\ v1(t) = 1111.11395418500251,\ v2(t) = 1563.06064783787088] \quad (26)$$

```
> dsol3(29.3);
```

$$[t = 29.3,\ v0(t) = 0.0289457959131091402,\ v1(t) = 1288.12295260664724,\ v2(t) = 1810.90106529687319] \quad (27)$$

Note that the reserve turns slightly negative very briefly during the last year of the policy.