This is the definition of the force of mortality for the G82M table.

```
> m := t ->
  0.0005+0.00007585775*10^(0.038*t) ;
```

$$m := t \to 0.0005 + 0.00007585775\ 10^{(0.038\ t)}$$

```
> evalf(0.00007585775*10^(0.038*40)/(0.038*ln
  (10)));
```

$$0.02870785022$$

Some useful definitions, which are selfexplanatory

```
> pa := t ->
  exp(-0.0005*t-.2870785022e-1*(10^(0.038*t)-
  1));
```

$$pa := t \to \mathbf{e}^{(-0.0005\ t - 0.02870785022\ (10^{(0.038\ t)} - 1))}$$

```
> evalf(pa(10));
```

$$0.9558469375$$

```
> evalf(0.00007585775*10^(0.038*45)/(0.038*ln
  (10)));
```

$$0.04446319552$$

```
> pb := t ->
  exp(-0.0005*t-.4446319552e-1*(10^(0.038*t)-
  1));
```

$$pb := t \to \mathbf{e}^{(-0.0005\,t - 0.04446319552\,(10^{(0.038\,t)} - 1))}$$

```
> evalf(0.00007585775*10^(0.038*50)/(0.038*ln
  (10)));
```

$$0.06886533613$$

```
> pc := t ->
  exp(-0.0005*t-.6886533613e-1*(10^(0.038*t)-
  1));
```

$$pc := t \to \mathbf{e}^{(-0.0005\,t - 0.06886533613\,(10^{(0.038\,t)} - 1))}$$

```
> qa := t -> 1-pa(t);
```

$$qa := t \to 1 - \mathrm{pa}(t)$$

```
> evalf( qa(10));
```

$$0.0441530625$$

```
> qb := t -> 1-pb(t);
```

$$qb := t \to 1 - \mathrm{pb}(t)$$

```
> qc := t -> 1-pc(t);
```

$$qc := t \to 1 - \mathrm{pc}(t)$$

Let us now save some copying and pasting with the next definition

```
> as := t -> pa(t)*pb(t)*qc(t)*m(45+t) +
  pa(t)*qb(t)*pc(t)*m(50+t) +
  pa(t)*qb(t)*pc(t)*m(40+t) +
  pa(t)*pb(t)*qc(t)*m(40+t) +
```

```
   qa(t)*pb(t)*pc(t)*m(50+t) +
   qa(t)*pb(t)*pc(t)*m(45+t);
```

$$as := t \to \mathrm{pa}(t)\,\mathrm{pb}(t)\,\mathrm{qc}(t)\,\mathrm{m}(45+t) + \mathrm{pa}(t)\,\mathrm{qb}(t)\,\mathrm{pc}(t)\,\mathrm{m}(50+t) + \mathrm{pa}(t)\,\mathrm{qb}(t)\,\mathrm{pc}(t)\,\mathrm{m}(40+t) + \mathrm{pa}(t)\,\mathrm{pb}(t)\,\mathrm{qc}(t)\,\mathrm{m}(40+t) + \mathrm{qa}(t)\,\mathrm{pb}(t)\,\mathrm{pc}(t)\,\mathrm{m}(50+t) + \mathrm{qa}(t)\,\mathrm{pb}(t)\,\mathrm{pc}(t)\,\mathrm{m}(45+t)$$

This is the assurance function

```
> evalf( Int( exp(-0.04*t)*as(t), t=0..20 ));
```

$$0.05929972908$$

This is the annuity with everybody alive

```
> evalf( Int( exp(-0.04*t)*pa(t)*pb(t)*pc(t),
  t=0..10 ));
```

$$7.579881161$$

This is an annuity with exactly two alive, so you have to add this to the previous one to get the annuity with at least two alive

```
> evalf( Int( exp(-0.04*t)*(pa(t)*pb(t)*qc(t)
  + pa(t)*qb(t)*pc(t) + qa(t)*pb(t)*pc(t)),
  t=0..10 ));
```

$$0.6357974115$$

Premium calculations for each case

```
> evalf( 1000000* .5929972908e-1/0.95);
```

$$62420.76745$$

```
> evalf(1000000*
  .5929972908e-1/(0.95*7.579881161));
```

$$8235.058852$$

```
> evalf(1000000*
  .5929972908e-1/(0.95*(7.579881161+.63579741
```

```
   15)));
```

7597.761635

With the next two integrals we calculate the probability that A was the survivor that got the money. Do the corresponding ones for B and C on yor own

```
> evalf( Int( as(t), t=0..20 ));
```

0.1074624122

```
> evalf( Int( pa(t)*pb(t)*qc(t)*m(45+t) +
  pa(t)*qb(t)*pc(t)*m(50+t), t=0..20 ));
```

0.05176547002

```
> evalf(.5176547002e-1/.1074624122);
```

0.4817076870

We now calculate the same probability given that the death was at time 15.

```
> evalf(as(15));
```

0.008758948496

```
> evalf(pa(15)*pb(15)*qc(15)*m(45+15) +
  pa(15)*qb(15)*pc(15)*m(50+15));
```

0.004216562335

```
> evalf(.4216562335e-2/.8758948496e-2);
```

0.4814005171

Now on to exercise 8. This is the probability that they will die within 10 years of each other; One of the integrals does not contribute much, but we should not forget it.

```
> evalf( Int(
  pa(t)*m(40+t)*(pb(t-10)-pb(t+10)), t=10..90
  ));
```

0.4132257656

```
> evalf( Int( pa(t)*m(40+t)*(1-pb(t+10)),
  t=0..10 ));
```

0.005839286277

```
> evalf(.4132257656+.5839286277e-2);
```

0.4190650519

And this is the expected value of the time between their deaths. Why this is correct? Think!

```
> evalf( Int( pa(t)*qb(t) + qa(t)*pb(t),
  t=0..90 ));
```

14.41788681

Finally, two ways to calculate the expected value of the interval between the first and the last death for 3 lives

```
> evalf(  Int( 1 - pa(t)*pb(t)*pc(t) -
  qa(t)*qb(t)*qc(t) , t=0..90 ));
```

21.74701188

```
> evalf(  Int(  pa(t)*pb(t)*qc(t) +
```

```
   pa(t)*qb(t)*pc(t) + qa(t)*pb(t)*pc(t) +
   qa(t)*qb(t)*pc(t) + qa(t)*pb(t)*qc(t) +
   pa(t)*qb(t)*qc(t) , t=0..90 ));
```

21.74701188

Thank you for your attention :-)

>