Let us start by defining the force of mortality

```
> m := t ->
  0.0005+0.00007585775*10^(0.038*t);
```

$$m := t \to 0.0005 + 0.00007585775\ 10^{(0.038\ t)}$$

This is the system of forward equations. The boundary conditions reflect that we assume that the 35 year old life (sorry about not giving you the age, let us keep it the same as in the origianl worksheet) is initially at 1.

```
> dsys1 :=
  {diff(p1(t),t)=0.1*p2(t)-(m(35+t)+0.1)*p1(t
  ),
  diff(p2(t),t)=0.1*p1(t)-(m(35+t)+0.108)*p2(
  t), p1(0)=1, p2(0)=0};
```

$$dsys1 := \{\, \mathrm{p1}(0) = 1, \frac{d}{dt}\mathrm{p1}(t) = 0.1\ \mathrm{p2}(t) - (0.1005 + 0.00007585775\ 10^{(1.330 + 0.038\ t)})\ \mathrm{p1}(t), \mathrm{p2}(0) = 0, \frac{d}{dt}\mathrm{p2}(t) = 0.1\ \mathrm{p1}(t) - (0.1085 + 0.00007585775\ 10^{(1.330 + 0.038\ t)})\ \mathrm{p2}(t)\,\}$$

Suppose we are interested in the various probabilities till time 60. We solve till time 60;

```
> dsol1 := dsolve(dsys1, numeric, range =
  0..60);
```

$$dsol1 := \mathbf{proc}(x\_rkf45)\ \ldots\ \mathbf{end\ proc}$$

Here are the probabilities at various times.

```
> dsol1(30);
```

$[t = 30., \mathrm{p1}(t) = 0.360015796122850062,$
$\mathrm{p2}(t) = 0.344265953426468984]$

```
> dsol1(25);
```

$[t = 25., \mathrm{p1}(t) = 0.404564844038087346,$
$\mathrm{p2}(t) = 0.383720132441347339]$

```
> dsol1(20);
```

$[t = 20., \mathrm{p1}(t) = 0.443184511410468274,$
$\mathrm{p2}(t) = 0.411108851607546344]$

```
> dsol1(15);
```

$[t = 15., \mathrm{p1}(t) = 0.483957690935715557,$
$\mathrm{p2}(t) = 0.422519177484097064]$

```
> dsol1(10);
```

$[t = 10., \mathrm{p1}(t) = 0.545234294520864916,$
$\mathrm{p2}(t) = 0.402831137137087902]$

```
> dsol1(5);
```

$[t = 5., \mathrm{p1}(t) = 0.674313963406491412,$
$\mathrm{p2}(t) = 0.305920768582001234]$

```
> dsol1(1);
```

$[t = 1., \mathrm{p1}(t) = 0.907359578172576842,$
$\mathrm{p2}(t) = 0.0900751441411961895]$

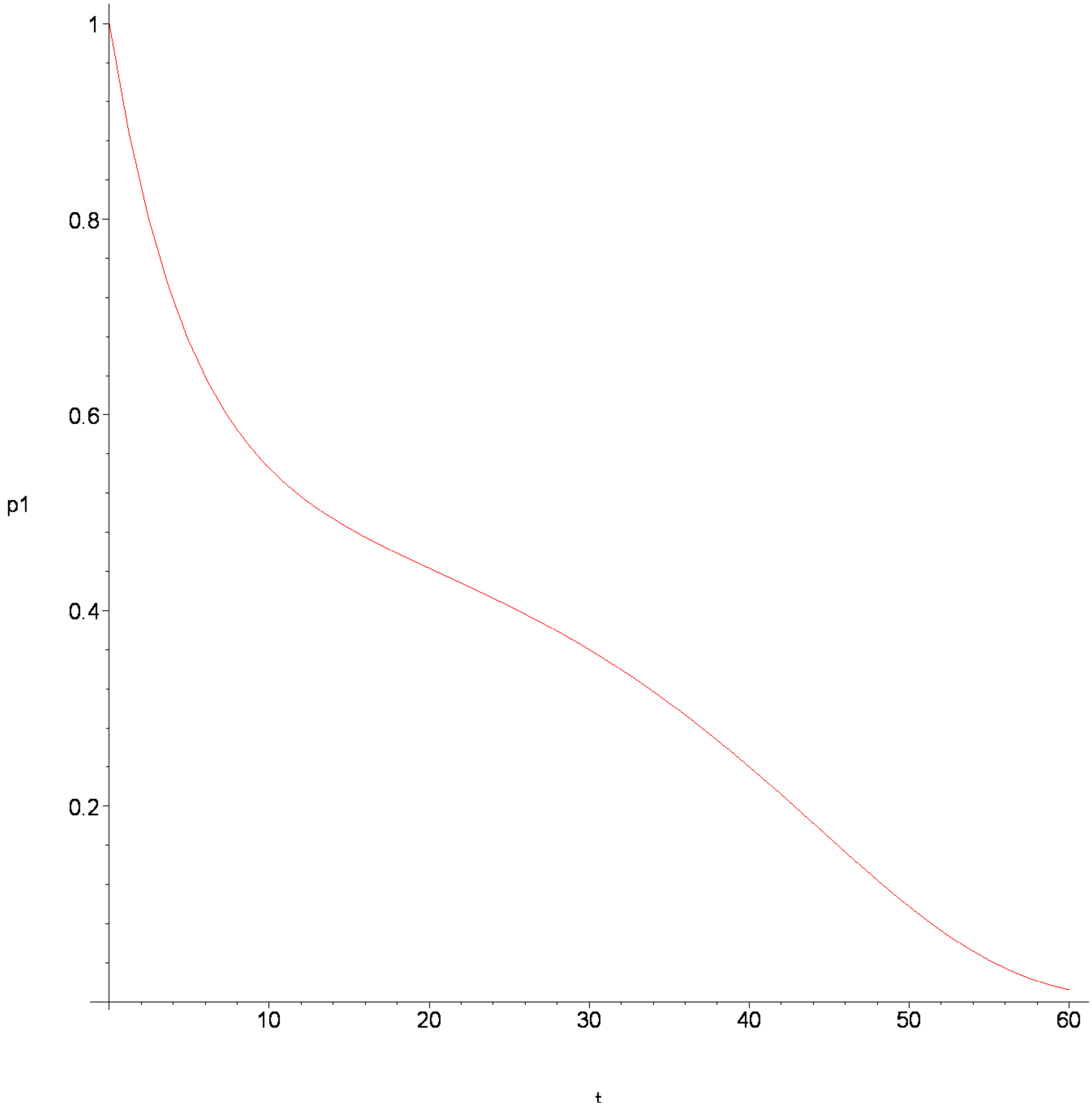
Let us plot

```
> with(plots);
Warning, the name changecoords has been redefined
```

[*animate*, *animate3d*, *animatecurve*, *arrow*, *changecoords*, *complexplot*, *complexplot3d*, *conformal*, *conformal3d*, *contourplot*, *contourplot3d*, *coordplot*, *coordplot3d*, *cylinderplot*, *densityplot*, *display*, *display3d*, *fieldplot*, *fieldplot3d*, *gradplot*, *gradplot3d*, *graphplot3d*, *implicitplot*, *implicitplot3d*, *inequal*, *interactive*, *listcontplot*, *listcontplot3d*, *listdensityplot*, *listplot*, *listplot3d*, *loglogplot*, *logplot*, *matrixplot*, *odeplot*, *pareto*, *plotcompare*, *pointplot*, *pointplot3d*, *polarplot*, *polygonplot*, *polygonplot3d*, *polyhedra_supported*, *polyhedraplot*, *replot*, *rootlocus*, *semilogplot*, *setoptions*, *setoptions3d*, *spacecurve*, *sparsematrixplot*, *sphereplot*, *surfdata*, *textplot*, *textplot3d*, *tubeplot*]

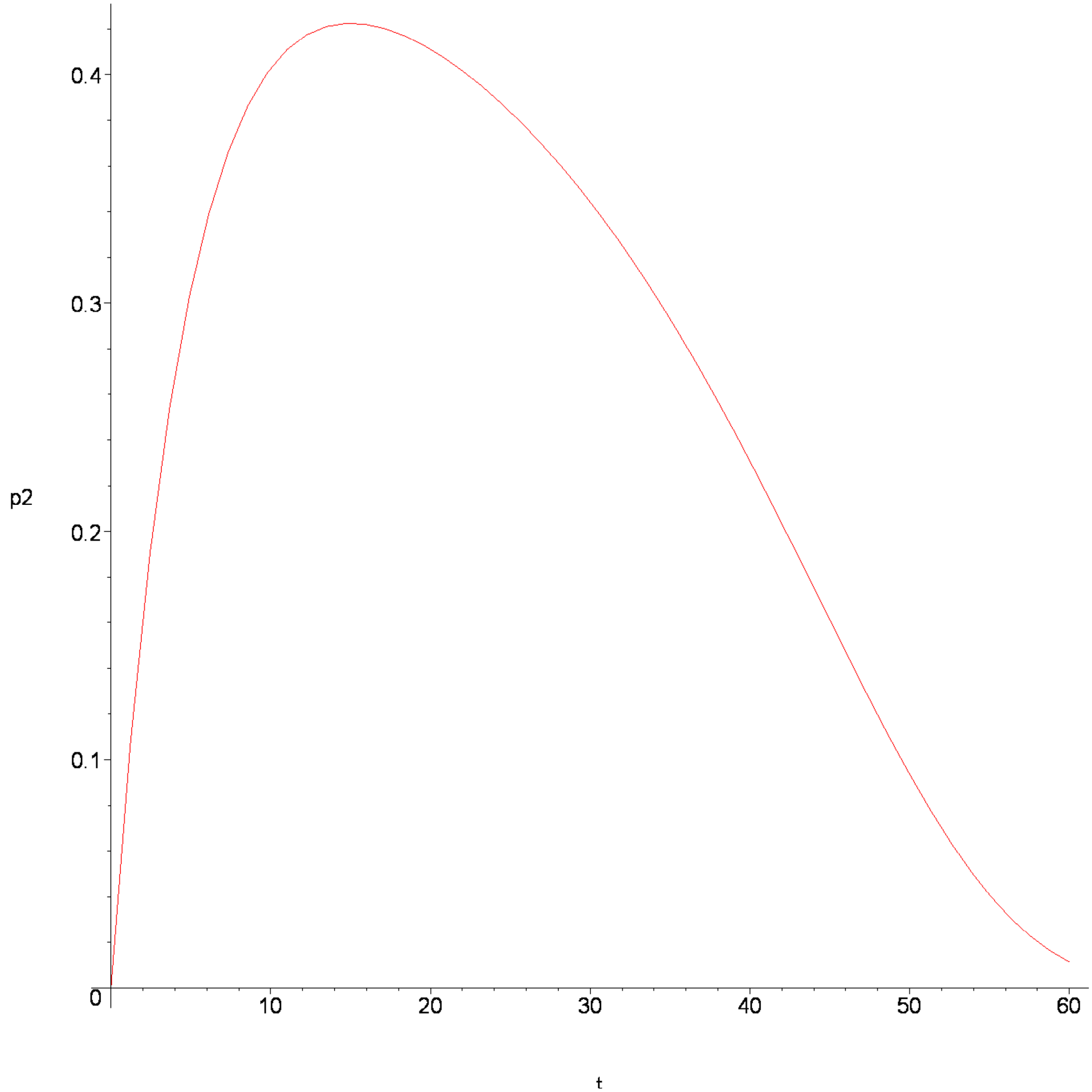
This is the plot of p1(t)

```
> odeplot(dsol1);
```



We can also plot p2(t)

```
> odeplot(dsol1, [t,p2(t)]);
```



Let us show that these ar te same as the ones in question 2

```
> evalf(0.00007585775*10^(0.038*35)/(0.038*ln
  (10)));
```

0.01853534490

```
> p := t ->
  exp(-0.0005*t-.1853534490e-1*(10^(0.038*t)-
  1));
```

$$p := t \rightarrow e^{(-0.0005\,t - 0.01853534490\,(10^{(0.038\,t)} - 1))}$$

```
> pp2 := t ->
  0.4996*(exp(-0.00392*t)-exp(-0.20408*t))*p(
  t);
```

$$pp2 := t \rightarrow 0.4996\,(e^{(-0.00392\,t)} - e^{(-0.20408\,t)})\,p(t)$$

```
> evalf(pp2(30));
```

$$0.3442658602$$

```
> evalf(pp2(10));
```

$$0.4028308176$$

They are the same. Let us now answer question 4

```
> evalf(.4028311371370879O2*(m(45)+0.008));
```

$$0.004991259492$$

```
> evalf(.5452342945208649l6*m(45));
```

$$0.002393824489$$

```
> evalf(.4991259492e-2/(.2393824489e-2+.49912
  59492e-2));
```

$$0.6758568359$$