

```
> m := t -> 0.0005+0.00007585775*10^(0.038*t) ;
```

$$m := t \rightarrow 0.0005 + 0.00007585775 \cdot 10^{(0.038t)} \quad (1)$$

```
> evalf(0.00007585775*10^(0.038*30)/(0.038*ln(10)));
0.01196742383
```

(2)

```
> p := t -> exp(-0.0005*t-0.01196742383*(10^(0.038*t)-1));
p := t -> e^(-0.0005t - 0.01196742383 (10^(0.038t) - 1))
```

(3)

```
> evalf(p(20));
```

0.9352906781

(4)

```
> evalf(p(30));
```

0.8451602610

(5)

We will now calculate annuities that we use in the typed solution

```
> deq1 := diff(v(t),t)-(0.04+m(30+t))*v(t)+1=0;
```

$$deq1 := \frac{d}{dt} v(t) - (0.0405 + 0.00007585775 \cdot 10^{(1.140 + 0.038t)}) v(t) + 1 = 0 \quad (6)$$

```
> tc1 := v(30)=0;
```

tc1 := v(30) = 0

(7)

```
> dsol1 := dsolve({deq1,tc1}, numeric, range=0..20) ;
```

dsol1 := proc(x_rkf45) ...end proc (8)

```
> dsol1(0);
```

[t=0., v(t)=16.8054148087885302]

(9)

```
> dsol1(10);
```

[t=10., v(t)=13.1676481415547162]

(10)

```
> dsol1(20);
```

[t=20., v(t)=7.91873959467779009]

(11)

Let us now calculate the integrals for part 3 by solving the differential equations.

```
> dsys1 := {diff(w1(t),t)=(m(30+t)+0.06)*w1(t)+1,diff(w2(t),t)=(m(30+t)+0.06)*w2(t)+m(30+t),diff(z1(t),t)=0.02*z1(t)+w1(t),diff(z2(t),t)=0.02*z2(t)+w2(t),w1(0)=0,w2(0)=0,z1(0)=0,z2(0)=0};
```

$$dsys1 := \left\{ \frac{d}{dt} z1(t) = 0.02 z1(t) + w1(t), \frac{d}{dt} z2(t) = 0.02 z2(t) \right. \quad (12)$$

$$+ w2(t), w1(0) = 0, w2(0) = 0, z1(0) = 0, z2(0) = 0, \frac{d}{dt} w1(t) = (0.0605$$

$$+ 0.00007585775 \cdot 10^{(1.140 + 0.038t)}) w1(t) + 1, \frac{d}{dt} w2(t) = (0.0605$$

$$+ 0.00007585775 \cdot 10^{(1.140 + 0.038t)}) w2(t) + 0.0005 + 0.00007585775 \cdot 10^{(1.140 + 0.038t)}) \}$$

```
> dsol1 := dsolve(dsys1, numeric, range = 0..30);
dsol1 := proc(x_rkf45) ...end proc
```

(13)

```
> dsol1(30);
[t = 30., w1(t) = 96.3653651041117456, w2(t) = 0.376066385217659228, z1(t) = 1147.62486828424540, z2(t) = 3.47117935646316766]
```

(14)

```
> dsol1(20);
[t = 20., w1(t) = 40.5534114503314386, w2(t) = 0.116618944781396880, z1(t) = 360.840870630485085, z2(t) = 0.859920444979915488]
```

(15)

```
> Let us check that the answer is correct by solving the system.
dsys2 := { diff(v(t), t) = 0.02 · v(t) − 0.02 · u(t) + 0.02, diff(u(t), t) = 0.0266 − m(30 + t) − .02 + (m(30 + t) + 0.06) · u(t), v(0) = 0, u(0) = 0 };
dsys2 := { v(0) = 0, u(0) = 0,  $\frac{d}{dt} u(t) = 0.0061 - 0.00007585775 \cdot 10^{(1.140 + 0.038t)} + (0.0605 + 0.00007585775 \cdot 10^{(1.140 + 0.038t)}) u(t), \frac{d}{dt} v(t) = 0.02 v(t) - 0.02 u(t) + 0.02 \}$ 
```

(16)

```
> dsol2 := dsolve(dsys2, numeric, range = 0..30);
dsol2 := proc(x_rkf45) ...end proc
```

(17)

```
> dsol2(30);
[t = 30., u(t) = 0.259944911962300296, v(t) = 0.740055951732377593]
```

(18)

```
> dsol2(20);
[t = 20., u(t) = 0.151033541454471565, v(t) = 0.461392121687459078]
```

(19)

```
>
```