We start with the usual assumptions

```
> m := t -> 0.0005+0.00007585775*10^(0.038*t)
  ;
```

$$m := t \rightarrow 0.0005 + 0.00007585775\ 10^{(0.038\ t)}$$

```
> evalf(0.00007585775*10^(0.038*40)/(0.038*ln
  (10)));
```

$$0.02870785022$$

```
> p := t ->
  exp(-0.0005*t-.2870785022e-1*(10^(0.038*t)-
  1));
```

$$p := t \rightarrow \mathbf{e}^{(-0.0005\ t - 0.02870785022\ (10^{(0.038\ t)} - 1))}$$

We now calculate the premium using the first order basis

```
> evalf( Int( exp(-0.04*t)*p(t)*m(40+t),
  t=0..20 ));
```

$$0.08519272119$$

```
> evalf(p(20)*exp(-0.04*20));
```

$$0.3881014187$$

```
> evalf(Int( exp(-0.04*t)*p(t), t=0..20 ));
```

$$13.16764650$$

```
> evalf(100000*(.8519272119e-1+.3881014187)/1
  3.16764650);
```

$$3594.371552$$

Let us check that Thiele's equation works OK as we will need it.

```
> deq1  := diff(v(t),t) = (0.04+m(40+t))*v(t)
```

```
- m(40+t)*100000 + 3594.371552;
```

$$deq1 := \frac{d}{dt}\mathrm{v}(t) = (0.0405 + 0.00007585775\ 10^{(1.520+0.038\,t)})\ \mathrm{v}(t) + 3544.371552 - 7.585775000\ 10^{(1.520+0.038\,t)}$$

```
> tc1    := v(20) = 100000;
```

$$tc1 := \mathrm{v}(20) = 100000$$

```
> dsol1 := dsolve({deq1,tc1}, numeric,
  range=0..20);
```

$$dsol1 := \textbf{proc}(x\_rkf45)\ \ldots\ \textbf{end proc}$$

```
> dsol1(0);
```

$$[t = 0.,\ \mathrm{v}(t) = -0.00429564135083637666]$$

```
> dsol1(20);
```

$$[t = 20.,\ \mathrm{v}(t) = 100000.]$$

We set out our equations as in normal exercise 21.

```
> dsys1 := {diff(v(t),t) =
  (0.04+m(40+t))*v(t) - m(40+t)*100000 +
  3594.371552,
  diff(w1(t),t)=-(0.06+m(40+t))*w1(t)+0.5*(w1
  (t)-w2(t)),
  diff(w2(t),t)=-(0.03+m(40+t))*w2(t)+0.5*(w2
  (t)-w1(t)),
  diff(ww1(t),t)=-(0.06-0.04)*w1(t)*v(t)+0.5*
  (ww1(t)-ww2(t)),
  diff(ww2(t),t)=-(0.03-0.04)*w2(t)*v(t)+0.5*
  (ww2(t)-ww1(t)),v(20)=100000,
  w1(20)=1,w2(20)=1,ww1(20)=0,ww2(20)=0};
```

$$dsys1 := \{\frac{d}{dt}\mathrm{v}(t) =$$

$$(0.0405 + 0.00007585775\ 10^{(1.520 + 0.038\ t)})\ \mathrm{v}(t) + 3544.371552$$

$$-\ 7.585775000\ 10^{(1.520 + 0.038\ t)},\ \mathrm{v}(20) = 100000,\ \mathrm{ww1}(20) = 0,$$

$$\mathrm{ww2}(20) = 0,$$

$$\frac{d}{dt}\mathrm{ww1}(t) = -0.02\ \mathrm{w1}(t)\ \mathrm{v}(t) + 0.5\ \mathrm{ww1}(t) - 0.5\ \mathrm{ww2}(t),$$

$$\frac{d}{dt}\mathrm{ww2}(t) = 0.01\ \mathrm{w2}(t)\ \mathrm{v}(t) + 0.5\ \mathrm{ww2}(t) - 0.5\ \mathrm{ww1}(t),$$

$$\mathrm{w1}(20) = 1,\ \mathrm{w2}(20) = 1,\ \frac{d}{dt}\mathrm{w1}(t) =$$

$$-(0.0605 + 0.00007585775\ 10^{(1.520 + 0.038\ t)})\ \mathrm{w1}(t) + 0.5\ \mathrm{w1}(t)$$

$$-\ 0.5\ \mathrm{w2}(t),\ \frac{d}{dt}\mathrm{w2}(t) =$$

$$-(0.0305 + 0.00007585775\ 10^{(1.520 + 0.038\ t)})\ \mathrm{w2}(t) + 0.5\ \mathrm{w2}(t)$$

$$-\ 0.5\ \mathrm{w1}(t)\}$$

```
> dsol1 := dsolve(dsys1, numeric, range =
  0..20);
```

$$dsol1 := \mathbf{proc}(x\_rkf45)\ \ldots\ \mathbf{end\ proc}$$

```
> dsol1(0);
```

$$[t = 0.,\ \mathrm{v}(t) = -0.771297459323250223\ 10^{-5},$$

$$\mathrm{w1}(t) = 2.90271930309045523,\ \mathrm{w2}(t) = 2.81694365408848491,$$

$$\mathrm{ww1}(t) = 6686.32936239311858,$$

$ww2(t) = 6418.78120015457080]$

We see that the prediction for the bonus is 6686.33. If the interest rate were 0.03 it would have been 6418.78.

We now have to calculate the accumulation of profit till time 5. Note that the equations are solved in a forward way.

```
> dsys2 := {diff(v(t),t) =
  (0.04+m(40+t))*v(t) - m(40+t)*100000 +
  3594.371552,
  diff(u(t),t)=(0.06+m(40+t))*u(t)+(0.06-0.04
  )*v(t), v(0)=0,u(0)=0};
```

$$dsys2 := \{\frac{d}{dt}\mathrm{v}(t) = (0.0405 + 0.00007585775\ 10^{(1.520 + 0.038\ t)})\ \mathrm{v}(t) + 3544.371552 - 7.585775000\ 10^{(1.520 + 0.038\ t)}, \mathrm{v}(0) = 0, \mathrm{u}(0) = 0, \frac{d}{dt}\mathrm{u}(t) = (0.0605 + 0.00007585775\ 10^{(1.520 + 0.038\ t)})\ \mathrm{u}(t) + 0.02\ \mathrm{v}(t)\}$$

```
> dsol2 := dsolve(dsys2, numeric, range =
  0..5);
```

$$dsol2 := \mathbf{proc}(x\_rkf45)\ \ldots\ \mathbf{end\ proc}$$

```
> dsol2(5);
```

$$[t = 5., \mathrm{u}(t) = 975.502325412271148, \mathrm{v}(t) = 18068.0820667158441]$$

We see that the accumulated bonus is 975.50. We now calculate the quantities we need from the large set of equations (no need to solve them again)

```
> dsol1(5);
```

$[t = 5., \mathrm{v}(t) = 18068.0820571398472,$
$\mathrm{w1}(t) = 2.27338041115803558, \mathrm{w2}(t) = 2.20620181003572169,$
$\mathrm{ww1}(t) = 6702.77707125526467,$
$\mathrm{ww2}(t) = 5289.36505208926246]$

```
> evalf(975.502325412271148*2.27338041115803558+6702.77707125526467);
```

$$8920.464948$$

So the prediction becomes 8920.46 (things went well over the first 5 years). Note that 6702.77 would be the prediction if nothing had been accumulated so far. Note that 6702.77>6686.33. Isn't this a bit strange? How can you explain it?

```
>
```