

An office issues 30-year unit-linked endowment assurance policies to lives aged 30. The policies are financed by a continuous premium payable at rate π . In order to calculate the premium the office makes the following assumptions. At any time $t(< n)$ a proportion $1 - \gamma_t$ of the premium is invested in a unit fund that is assumed to grow at a force of interest of 0.04. A proportion γ_t is allocated to a cash fund that grows at a constant force of interest 0. At time 30 or on earlier death the policyholder will receive the accumulated amount of the fund or a guaranteed sum 1000000, whichever is larger. γ_t is calculated in such a way that

$$\gamma_t = \mu_{x+t}(b - U(t))_+ + 0.08\pi$$

where $U(t)$ is the value of the unit fund at time t and is the force of mortality. There is also the possibility of surrenders which occur independently of deaths and with rate $0.02 - \mu_{x+t}$. (You may assume this is positive). Surrenders are entitled to their share of the unit fund (without guarantees) but not to any of the cash fund.

1. Calculate π using MAPLE and the usual force of mortality.
2. Provide a prediction for the accumulation of the unit fund at time 20 using the assumptions of the question.
3. Repeat 1 and 2 using

$$\gamma_t = (\mu_{x+t} + 0.02)(b - U(t))_+.$$