

Solutions to Extra Exercise 2

2nd Oct 2021

The life is aged 50.5 now.

1. Discrete case

(i) Prospective:

$$\begin{aligned}
 & EPV(benefits) - EPV(premiums) \\
 = & 30000 {}_{14.5}p_{50.5} e^{-14.5 \times 0.04} \bar{a}_{65} + 100000 \bar{A}_{50.5:\overline{19.5}|}^1 - 5126 \sum_{t=1}^{14} e^{-0.04t} {}_tp_{50.5} \\
 = & 30000 \times 0.8260 \times 0.5599 \times 10.5705 + 100000 \times 0.1257 - 5126 \times 9.8007 \\
 = & 108990
 \end{aligned}$$

(ii) Retrospective:

$$\begin{aligned}
 & \text{Accumulation of premiums - benefits} \\
 = & 5126 \sum_{t=0}^{15} \frac{e^{0.04(15-t)}}{{}_tp_{35.5}} - 100000 \int_0^{15} \mu_{35.5+t} e^{\int_t^{15} \mu_{35.5+s} + 0.04 ds} \\
 = & 5126 \sum_{t=0}^{15} \frac{e^{0.04(15-t)}}{{}_tp_{35.5}} - 100000 \int_0^{15} \mu_{35.5+t} e^{\int_t^{15} \mu_{35.5+s} + 0.04 ds} \\
 = & 5126 \times 22.8333 - 100000 \times 0.080432 \\
 = & 109000
 \end{aligned}$$

2. Continuous case

(i) Prospective:

$$\begin{aligned}
 & 30000 {}_{14.5}p_{50.5} e^{-14.5 \times 0.04} \bar{a}_{65} + 100000 \bar{A}_{50.5:\overline{19.5}|}^1 - 5284 \bar{a}_{50.5:\overline{14.5}|} \\
 = & 30000 \times 0.826 \times 0.5599 \times 10.5705 + 100000 \times 0.1257 - 5284 \times 10.2956 \\
 = & 104827
 \end{aligned}$$

(ii) Retrospective:

$$\begin{aligned}
 & 5284 \int_0^{15} e^{0.04(15-t)} e^{\int_t^{15} \mu_{35.5+u} du} dt - 100000 \int_0^{15} \mu_{35.5+t} e^{\int_t^{15} (\mu_{35.5+s} + 0.04) ds} dt \\
 = & 5284 \times 21.3629 - 100000 \times 0.080432 \\
 = & 104838
 \end{aligned}$$

The reserves are the same for both cases.

2nd April 2041

The reserves for both the discrete and continuous case are the same, since at age 70, all the premiums have already been paid.

(i) Prospective:

$$\begin{aligned} 30000\bar{a}_{70} &= 30000 \times 8.8997 \\ &= 266991 \end{aligned}$$

(ii) Retrospective: (by principle of equivalence, we write the premiums as sum of the annuity and death benefits)

$$\begin{aligned} & 30000\bar{a}_{65}e^{0.04 \times 5}e^{\int_0^5 \mu_{65+t}dt} - 30000 \int_0^5 e^{\int_t^5 \mu_{65+u}du}e^{0.04(5-t)}dt + \text{death benefits} - \text{death benefits} \\ &= 30000 \times 10.5705 \times 1.2214 \times 1.1536 - 30000 \times 5.9947 \\ &= 266976 \end{aligned}$$

So the reserves are the same.