

This is the definition of the force of mortality for the G82M table.

```
> m := t ->  
0.0005+0.00007585775*10^(0.038*t) ;
```

$$m := t \rightarrow 0.0005 + 0.00007585775 \cdot 10^{(0.038 t)}$$

This is the survival function for a life aged 30

```
> evalf(0.00007585775*10^(0.038*30)/(0.038*ln  
(10)));
```

$$0.01196742383$$

```
> p := t ->  
exp(-0.0005*t-0.01196742383*(10^(0.038*t)-1  
));
```

$$p := t \rightarrow e^{(-0.0005 t - 0.01196742383 (10^{(0.038 t)} - 1))}$$

This is a continuous 30 year annuity for a life aged 30.

```
> evalf( Int( exp(-0.05*t)*p(t), t=0..30 ));
```

$$14.99247514$$

The endowment assurance is

```
> evalf(1-0.05*14.99247514);
```

$$0.2503762430$$

The premium is

```
> evalf(.2503762430/14.99247514);
```

0.01670012727

Thiele's equation

```
> deq1 := diff(v(t), t) -  
      (0.05+m(30+t))*v(t) - .1670012727e-1 +  
      m(30+t)= 0;
```

$$deq1 := \left(\frac{d}{dt} v(t) \right)$$

$$- (0.0505 + 0.00007585775 \cdot 10^{(1.140 + 0.038 t)}) v(t)$$

$$- 0.01620012727 + 0.00007585775 \cdot 10^{(1.140 + 0.038 t)} = 0$$

```
> tc1 := v(30) = 1;
```

$$tc1 := v(30) = 1$$

```
> dsol1 := dsolve({deq1, tc1}, numeric,  
      range=0..30);
```

dsol1 := proc(*x_rkf45*) ... end proc

```
> dsol1(0);
```

$$[t = 0., v(t) = -0.572022031616881144 \cdot 10^{-7}]$$

```
> dsol1(5);
```

$$[t = 5., v(t) = 0.0850542441844581165]$$

```
> dsol1(10);
```

$$[t = 10., v(t) = 0.191746596617043186]$$

```
> dsol1(20);
```

$$[t = 20., v(t) = 0.495288131004382626]$$

```
> dsol1(25);
```

$$[t = 25., v(t) = 0.712772498634511531]$$

```
> dsol1(30);
```

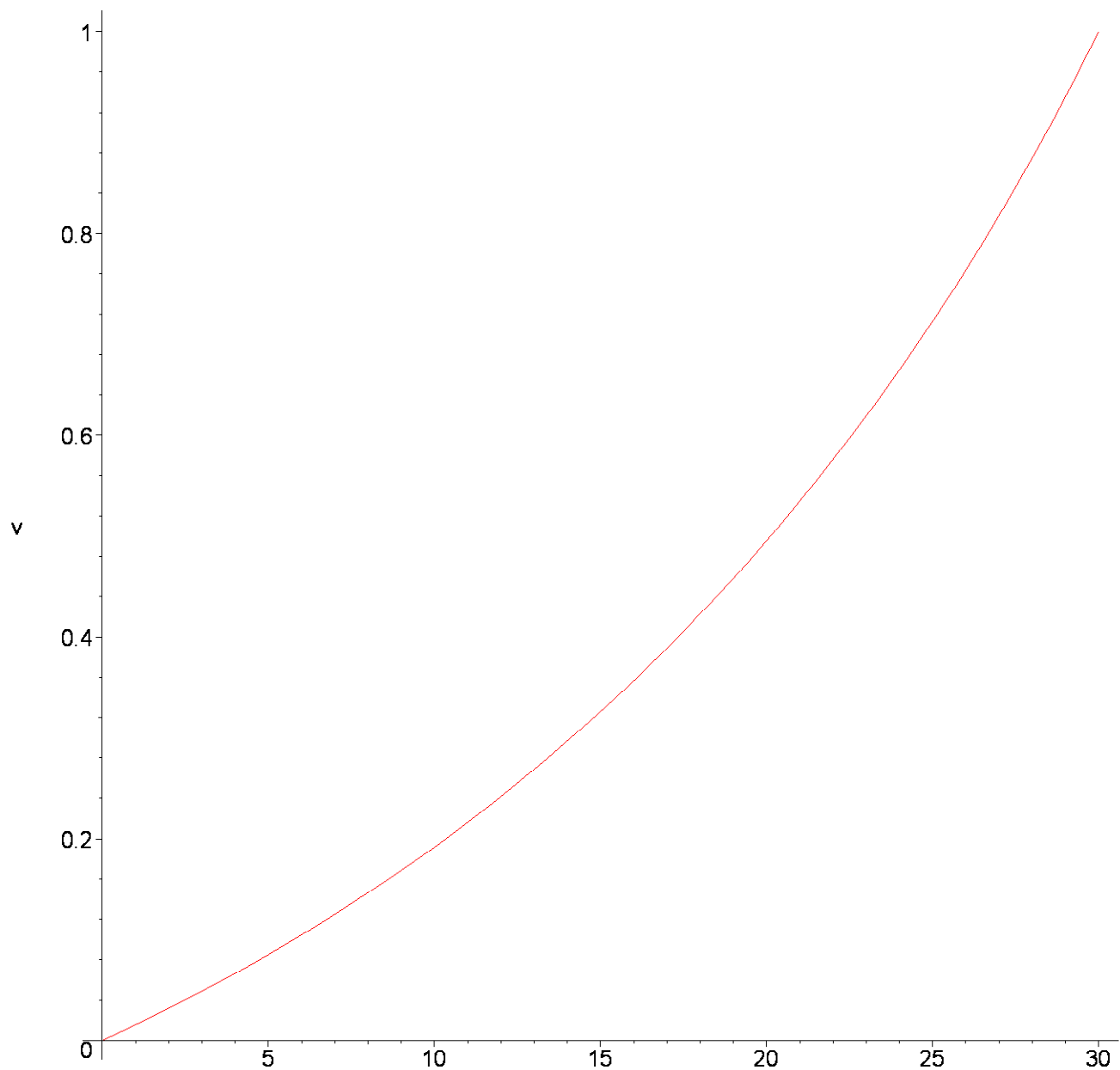
$$[t = 30., v(t) = 1.]$$

```
> with(plots);
```

Warning, the name `changecoords` has been redefined

```
[animate, animate3d, animatecurve, arrow, changecoords,  
complexplot, complexplot3d, conformal, conformal3d,  
contourplot, contourplot3d, coordplot, coordplot3d, cylinderplot,  
densityplot, display, display3d, fieldplot, fieldplot3d, gradplot,  
gradplot3d, graphplot3d, implicitplot, implicitplot3d, inequal,  
interactive, listcontplot, listcontplot3d, listdensityplot, listplot,  
listplot3d, loglogplot, logplot, matrixplot, odeplot, pareto,  
plotcompare, pointplot, pointplot3d, polarplot, polygonplot,  
polygonplot3d, polyhedra_supported, polyhedraplot, replot,  
rootlocus, semilogplot, setoptions, setoptions3d, spacecurve,  
sparsematrixplot, sphereplot, surfdata, textplot, textplot3d,  
tubeplot]
```

```
> odeplot(dsol1);
```



Now for the expenses case. The following calculations yield the premium (explained in class)

```
> evalf( Int( exp(-0.045*t)*p(t), t=0..30 ));
15.85986176
> evalf(p(30)*exp(-0.045*30));
0.2190995663
> evalf(1-0.045*15.85986176);
```

0.2863062208

```
> evalf((0.004+1.002*.2863062208-0.0019*.2190  
995663)/15.85986176);
```

0.01831431751

```
> evalf((.1831431751e-1+0.0001)/0.99);
```

0.01860032072

Thiele's equation

```
> deq2 := diff(v(t),t) -  
(0.045+m(30+t))*v(t) - .1831431751e-1 +  
1.002*m(30+t)= 0;
```

$$deq2 := \left(\frac{d}{dt} v(t) \right)$$

$$- (0.0455 + 0.00007585775 \cdot 10^{(1.140 + 0.038 t)}) v(t) \\ - 0.01781331751 + 0.00007600946550 \cdot 10^{(1.140 + 0.038 t)} = 0$$

```
> tc2 := v(30) = 1.0001;
```

$$tc2 := v(30) = 1.0001$$

```
> dsol2 := dsolve({deq2,tc2}, numeric,  
range=0..30);
```

dsol2 := proc(x_rkf45) ... end proc

Note that the initial reserve must be -0.004

```
> dsol2(0);
```

$$[t = 0., v(t) = -0.00400005510669848574]$$

```
> dsol2(30);
```

$$[t = 30., v(t) = 1.00010000000000]$$

```
> dsol2(5);
```

```
>
```

```
[  
    [  $t = 5.$ ,  $v(t) = 0.0879686078256026184$  ]  
> dsol2(10) ;  
    [  $t = 10.$ ,  $v(t) = 0.200737660553129554$  ]  
> dsol2(15) ;  
    [  $t = 15.$ ,  $v(t) = 0.339186212109453654$  ]  
> dsol2(20) ;  
    [  $t = 20.$ ,  $v(t) = 0.510154601859010071$  ]  
> dsol2(25) ;  
    [  $t = 25.$ ,  $v(t) = 0.724274783633621410$  ]  
> dsol2(30) ;  
    [  $t = 30.$ ,  $v(t) = 1.00010000000000$  ]  
> odeplot(dsol2) ;
```

