Istanbul Workshop on Mathematical Finance

Marmara Pera Hotel, Istanbul

Abstracts

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- **Philip Protter, Cornell University**

  Asset Pricing Theory with Bubbles:

  Asset pricing theory is often treated as though the price process under the risk neutral measure is a martingale. This is misleading, since when the price process is a strict local martingale, it may correspond to a bubble. In this series of lectures we will introduce financial asset pricing theory, explain risk neutral measures and self-financing strategies, and show the fundamental role played by bubbles. We will give examples of strict local martingales, and discuss how to generate them at will. Strict local martingales are a peculiarity of continuous time processes, and this illustrates the inadequacy of a discrete time theory. We will illustrate an effect of bubbles by showing Merton's "No Early Exercise" theorem for American call options fails, and give the striking example of the inverse Bessel process.

  We will explain issues of liquidity and supply curves, and show how liquidity problems alone can cause what we call a liquidity bubble, a different kind of bubble than a local martingale bubble. Finally, we will discuss how one might go about trying to detect whether or not one is currently in a bubble, which is a topic for future research.

  The course will draw on recent research of the expositor and collaborators, including Umut Çetin, Robert Jarrow, Soumik Pal, Kazuhiro Shimbo, and Alexandre Roch.

  Prerequisites: Knowledge of martingales and stochastic integration

- **Peter Bank, Technische Universität Berlin**

  A large investor trading at market indifference prices:

  We consider a financial market where a finite number of market makers quote prices for a given security. The market makers rehedge the acquired positions among themselves so as to keep the allocation of risk in a Pareto-optimum. We show how this amounts to a dynamic model for trades with permanent price impact and discuss how the implied continuous-time strategies relate to the Black-Scholes hedges.

(Joint work with Dmitry Kramkov).

- **Bruno Bouchard, Universite Paris-Dauphine**

  Weak Dynamic Programming Principle for Viscosity Solutions

  We prove a weak version of the dynamic programming principle for standard stochastic control problems and mixed control-stopping problems, which avoids the technical difficulties related to the measurable selection argument. In the Markov case, our result is tailor-maid for the derivation of the dynamic programming equation in the sense of viscosity solutions.
  This joint work with Nizar Touzi, Ecole Polytechnique.

- **Luciano Campi, Universite Paris Dauphine**

  Utility maximization with proportional transaction costs

  We present an optimal investment theorem for a currency exchange model with random and possibly discontinuous proportional transaction costs. The investor's preferences are represented by a smooth, multivariate utility function, allowing for simultaneous consumption of any prescribed selection of the currencies at a given terminal date. We prove the existence of an optimal portfolio process under the assumption of asymptotic satiability of the value function. Sufficient conditions for asymptotic satiability of the value function include reasonable asymptotic elasticity of the utility function, or a growth condition on its dual function. We show that the portfolio optimization problem can be reformulated in terms of maximization of a terminal liquidation utility function, and that both problems have a common optimizer. The talk is based on a joint paper with M. Owen.

- **Rene Carmona, Princeton University**

  Mathematics of the Emissions Markets

  The talk is concerned with the mathematical theory of the cap-and-trade schemes touted as the most efficient way to curb Green House Gas emissions. After reviewing recent results on price formation based on economic equilibrium theory, we demonstrate the desirable features of alternative allocations schemes on case studies based on the Texas and Japan electricity markets. If time permits, we will introduce a simple reduced form model which can be calibrated to existing allowance price data, and used to price options on forward allowance prices. This recent work was motivated by the increasing liquidity of the European Union ETS and the prospect of a unique federal cap-and-trade scheme in the US.

- **Christa Cuchiero, Vienna Institute of Finance**

  Polynomial processes and applications to option pricing

  We introduce a class of Markov stochastic processes called m-polynomial, for which the calculation of (mixed) moments up to order m only requires the computation of matrix exponentials. This class

contains affine processes, Feller processes with quadratic squared diffusion coefficient, as well as Lévy-driven SDEs with affine vector fields. Thus, many popular models such as the classical Black-Scholes, exponential Lévy or affine models are covered by this setting. The applications range from statistical GMM estimation to option pricing. For instance, the efficient and easy computation of moments can successfully be used for variance reduction techniques in Monte Carlo simulations.

- **Romuald Elie, Universite Paris-Dauphine**

  Quantile hedging and optimal control under stochastic target constraints

  We consider the problem of finding the minimal initial data of a controlled process which guarantees to reach a controlled target with a given probability of success or, more generally, with a given level of expected loss. By suitably increasing the state space and the controls, we show that this problem can be converted into a stochastic target problem, i.e. find the minimal initial data of a controlled process which guarantees to reach a controlled target with probability one. Unlike the existing literature on stochastic target problems, our increased controls are valued in an unbounded set. In this paper, we provide a new derivation of the dynamic programming equation for general stochastic target problems with unbounded controls, together with the appropriate boundary conditions. These results are applied to the problem of quantile hedging in financial mathematics, and are shown to recover the explicit solution of Follmer and Leukert. We then consider the problem of miximizing a utility function under this type of quantile constraint. The previous study allows to characterize the domain in which the value fuction lies and we provide an Hamilton-Jacobi-Bellman representation of the associated value function. Contrary to standard state constraint problems, the domain is not given a-priori and we do not need to impose conditions on its boundary.

- **Nicole El Karoui, Ecole Polytechnique**

  Dynamic forward utility and its volatility

  The classical theory of expected utility from terminal wealth yields to selecte optimal portfolio strongly depending of the horizon. In a variety of applications, for instance in indifference pricing, the investement horizon and the maturities of the claims do not coincide.This misalignment might cause price discrepancies. A new concept of dynamic forward utility has been intoduced by M.Musiela and T.Zariphopoulo. These utilities evolve forward in time adn are generated for future times via a self-generating criterion. Moreover a set of given portfolios is given as describing the investement universe. Comptatibility condition constraint the class of forward utilities. We studies in detail these utilities from the stochastic PDE point of view and the change of numeraire, putting special attention at the role of volatiliy of forward utilities. Both, primal and dual point of view are studied.
  This work is by N.El Karoui and M.M'Rad.

- **Arash Fahim, Ecole Polytechnique and Sharif University of Technology**

  A probabilistic numerical method for fully nonlinear parabolic PDEs

We consider the probabilistic numerical scheme for fully nonlinear PDEs suggested in [1], and show that it can be introduced naturally without appealing to the theory of backward stochastic differential equations. Our first main result provides the convergence of the discrete-time approximation and derives a bound on the discretization error in terms of the time step. An explicit implementable scheme requires to approximate the conditional expectation operators involved in the discretization. This induces a further Monte Carlo error. Our second main result is to prove the convergence of the latter approximation scheme, and to derive an upper bound on the approximation error. Numerical experiments are performed for the approximation of the solution of the mean curvature flow equation in dimensions two and three, and for two and five-dimensional (plus time) fully-nonlinear Hamilton-Jacobi-Bellman equations arising in the theory of portfolio optimization in financial mathematics. Finally we discuss the generalization of this method to nonlocal parabolic PDEs.
This is joint work with Nizar Touzi, Ecole Polytechnique.

References

[1] P. Cheridito, H.M. Soner, N. Touzi, N. Victoir; "Second Order Backward Stochastic Differential Equations and Fully Non-Linear Parabolic PDEs" Communications on Pure and Applied Mathematics, Volume 60, Issue 7, Date: July 2007, Pages: 1081-1110.

- **Hans Föllmer, Humboldt Universität zu Berlin**

  Dynamic risk assessment: martingale aspects of time-consistency

  We consider dynamic convex risk measures, both for fixed terminal positions and for cash-flows, and we discuss various notions of their time-consistency in terms of martingale properties. This will be based on joint work with Beatrice Acciaio and Irina Penner.

- **Pavel Gapeev, London School of Economics**

  Discounted optimal stopping for diffusions: free-boundary versus martingale approach

  The free-boundary and martingale approach are competitive methods of solving discounted optimal stopping problems for one-dimensional regular diffusion processes on the infinite time interval.

  We provide a missing link between these two approaches, showing their equivalence, by decomposing the reward process into a product of a martingale and a gain function of the stopping boundaries for the underlying process. We illustrate our result on several examples related to the rational valuation of perpetual American options. This talk will be based on joint work with Hans Rudolf Lerche from the University of Freiburg, Germany.

- **Lane Hughston, Imperial College**

  Modelling Information Flows in Financial Markets

The major determiner of price changes is "new information". When a new piece of information circulates in a financial market (whether true, partly true, misleading, or bogus), the prices of related assets will be adjusted in response, and will move again when the information is updated. The importance of the role of information is clear on an intuitive basis, but how do we actually model the flow of information? What is the information "about"? In this talk I discuss some of the issues involved in modelling the flow of information in financial markets, and I present some elementary models for information in various situations. Some applications to the pricing of various types of financial products will be indicated. In particular I shall look at the problem of correlation modelling, an issue where a back-to-the-basics approach is clearly needed. Finally, I shall make a few remarks about statistical arbitrage strategies, and about price formation in inhomogeneous markets. [Based on work carried out in collaboration with D. Brody, A. Macrina, E. Hoyle, and others.]

- **Monique Jeanblanc, Universite d'Evry**

  What happens after a default: the conditional density approach

  With N. EL KAROUI and Y. JIAO

  We present a general model for default time, making precise the role of the intensity process, and showing that this process allows for a knowledge of the conditional distribution of the default only "before the default". This lack of information is crucial while working in a multi-default setting. In a single default case, the knowledge of the intensity process does not allow to compute the price of defaultable claims, except in the case where immersion property is satisfied. The density process will give a full characterization of the links between the default time and the reference filtration, in particular "after the default time". We also investigate the description of martingales in the full filtration in terms of martingales in the reference filtration, and the impact of Girsanov transformation on the density process. We show that one can construct different models where the default time admits the same intensity process.

- **Anis Matoussi, Universite du Maine**

  Robust utility maximization problem from terminal wealth and consumption : BSDE approach

  We study a stochastic control problem in the context of utility maximization under model uncertainty. The problem is formulated as a $\sup - \inf$, sup over strategies π and consumption c and inf over the set of models (measures) $\mathcal{Q}$.

  For the minimization problem, we showed in [1] that there exists a unique optimal measure $\mathbb{Q}^*$ equivalent to the reference measure $\mathbb{P}$. Moreover, in the context of continuous filtration, we characterize the dynamic value process of our stochastic control problem as the unique solution of a generalized backward stochastic differential equation with a quadratic driver. In [4], we extend first this result in a discontinuous filtration. Moreover, we obtain a comparison theorem and a regularity properties for the associated generalized BSDE with jumps, which are the key points in our approach, in order

to solve the utility maximization problem over terminal wealth and consumptions. This comparison theorem was first obtained in [3] for the continuous generalized BSDE. Our results extend the both earlier works by El Karoui et al [2] and Skiadas [5].

References

[1] Bordigoni G., Matoussi,A., Schweizer, M. : A Stochastic control approach to a robust utility maximization problem. *F. E. Benth et al. (eds.), Stochastic Analysis and Applications. Proceedings of the Second Abel Symposium, Oslo, 2005*, *Springer*, *125-151 (2007)* .

[2] El Karoui, N. Peng, S., Quenez, M.-C. : A dynamic maximum principle for the optimization of recursive utilities under constraints. *Annals of Applied Probability* **11**, *664-693 (2001).*

[3] Faidi, W., Matoussi, A., Mnif, M. : Maximization of Stochastic Differential Utility : A dynamic maximum principle appraoch. forthcoming paper.

[4] Jeanblanc, M., Matoussi, A., Ngoupeyou, A. : Robust utility maximization from terminal wealth and consumption in a discontinuous filtration. forthcoming paper.

[5] Skiadas C. : Robust control and recursive utility. *Finance and Stochastics* **7**, *475- (2003).*

- **Ashkan Nikeghbali, University of Zurich**

  Last passage times in financial modeling and some related problems

  Recently, Madan, Roynette and Yor have been able to express the price of a European put option in terms of some last passage times. The goal of this talk is to provide a general framework in which we explain this formula and to see some further connections with the draw-down process. We shall also see how these results are related to some penalization problems.

- **Antonis Papapantoleon, Vienna University of Technology**

  A new approach to LIBOR modeling

  LIBOR market models are the favorite models of practitioners for the pricing of interest rate derivatives, however they suffer from severe intractability problems due to the random terms that enter the SDEs during the construction of the model. As a result, if the driving process is continuous then caplets can be priced in closed form, but not swaptions or other multi-LIBOR products; in case the driving process has jumps, then even caplets cannot be priced in closed form. In both cases, the calibration of the model to cap and swaption market data is very difficult and requires some short of approximation (e.g. "frozen drift" approximation). On the other hand, modeling forward prices produces a very tractable model, but negative LIBOR rates can occur, which contradicts any economic intuition.

  In this work we propose a new approach to modeling LIBOR rates based on affine factor processes. We construct suitable martingales that stay greater than one for all times, utilizing the Markov prop-

erty of affine processes. Then, we model LIBOR rates in a framework that produces positive LIBOR rates in an analytically tractable model; in particular, LIBOR rates have affine stochastic dynamics under *any* forward measure. Hence, this model unifies the advantages of LIBOR market models and forward price models. We derive Fourier transform valuation formulas for caplets and swaptions, hence the calibration of the model is very easy. Furthermore, when the driving process is the CIR process, closed form valuation formulas – using the χ^2-distribution function – are derived for caps and swaptions.

This talk is based on a joint article with Martin Keller-Ressel and Josef Teichmann [1].

References

[1] M. Keller-Ressel, A. Papapantoleon, and J. Teichmann, *A new approach to LIBOR modeling*. Preprint, `arXiv/0904.0555`, 2009.

- **Huyen Pham, Universite Paris VII**

  Optimal investment with counterparty risk: a default-density modeling approach

  We consider a financial market with a stock exposed to a counterparty risk inducing a drop in the price, and which can still be traded after this default time. We use a default-density modeling approach, and address in this incomplete market context the expected utility maximization from terminal wealth. We show how this problem can be suitably decomposed in two optimization problems in complete market framework: an after-default utility maximization and a global before-default optimization problem involving the former one. These two optimization problems are solved explicitly, respectively by duality and dynamic programming approaches, and provide a fine understanding of the optimal strategy. We give some numerical results illustrating the impact of counterparty risk on optimal trading strategies, in particular with respect to the Merton portfolio selection problem.

- **Thorsten Rheinländer, London School of Economics**

  Hedging survivor bonds with mortality-linked securities

  Recently there have been quite some transactions reported in derivatives written on some longevity index. We propose a new instrument, a kind of mortality bond, to hedge the digital payoff of a survivor bond. To calculate the optimal hedging portfolio, some issues about martingale representations with respect to different filtrations have to be resolved. Moreover, we discuss some general features of so-called new markets, and study several optimal martingale measures in our context.
  This is joint work with Francesca Biagini, Munich.

- **Anja Richter, Humboldt Universität zu Berlin**

  Differentiability of martingale driven BSDE and application to hedging in incomplete markets

  In this talk we consider quadratic growth BSDE driven by continuous local martingales. First we derive the Markov property of a forward-backward system when the driving martingale is a strong Markov process. Then we establish the differentiability of a FBSDE with respect to the initial value of its forward component. It enables us to obtain the main result of this talk that is to describe the control process of the BSDE in terms of a differential operator of the value process and the diffusion coefficient of the forward process. This formula generalizes the results obtained by several authors in the Brownian setting, designed to represent the optimal delta hedge in the context of cross hedging non-tradable derivatives that generalizes the derivative hedge in the Black-Scholes model. It involves Malliavin's calculus which is not available in the general martingale setting. Consequently, we propose a new method based on stochastic calculus techniques. This is a joint work with Peter Imkeller and Anthony Rveillac.

- **Chris Rogers, Cambridge University**

  Optimal and robust contracts for a risk-constrained principal

  The theory of risk measurement has been extensively developed over the past ten years or so, but there has been comparatively little effort devoted to using this theory to inform portfolio choice. One theme of this paper is to study how an investor in a conventional log-Brownian market would invest to optimize expected utility of terminal wealth, when subjected to a bound on his risk, as measured by a coherent law-invariant risk measure. Results of Kusuoka lead to remarkably complete expressions for the solution to this problem.

  The second theme of the paper is to discuss how one would actually manage (not just measure) risk. We study a principal/agent problem, where the principal is required to satisfy some risk constraint. The principal proposes a compensation package to the agent, who then optimises selfishly ignoring the risk constraint. The principal can pick a compensation package that induces the agent to select the principal's optimal choice. We consider two possibilities: firstly, that the principal chooses a contract which is cheapest subject to satisfying the agent's participation constraint; and secondly, a robust contract which perfectly aligns the objectives of principal and agent. The two typically differ little in price, though their form can look surprisingly different.

- **Walter Schachermayer, University of Vienna**

  The fundamental theorem of asset pricing for continuous processes under small transaction costs

  A version of the fundamental theorem of asset pricing is proved for continuous asset prices with small proportional transaction costs. Equivalence is established between: (a) the absence of arbitrage with general strategies for arbitrarily small transaction costs $\varepsilon > 0$, (b) the absence of free lunches with bounded risk for arbitrarily small transaction costs $\varepsilon > 0$, and (c) the existence of ε-consistent price

systems – the analogue of martingale measures under transaction costs – for arbitrarily small $\varepsilon > 0$. The proof proceeds through an explicit construction, as opposed to the usual separation arguments. The paper concludes comparing numeraire-free and numeraire-based notions of admissibility, and the corresponding martingale and local martingale properties for consistent price systems.

- **Martin Schweizer, ETH Zurich**

  Convexity techniques for BSDEs from utility indifference valuation

  One popular approach to the valuation of contingent claims in incomplete financial markets is via exponential utility indifference. The value process for the corresponding stochastic control problem can often be described by a backward stochastic differential equation (BSDE). This is in many cases very useful for proving theoretical properties; but actually solving these equations to obtain more explicit results is difficult, especially if the underlying asset is multidimensional. With the goal of obtaining more information, we therefore study BSDE transformations that allow us to derive upper and/or lower bounds, in terms of solutions of other BSDEs, that can be computed more explicitly. These ideas and techniques may also be of independent interest for theoretical BSDE studies. This is joint work with Christoph Frei and Semyon Malamud.

- **Jianfeng Zhang, University of Southern California**

  Laws of Large Numbers for Mean Number of Correlated Defaults

  We consider a model in which the default times of multiple firms are correlated by their dependence on the number of past defaults, in addition to their dependence on common factors. We characterize the limiting process for the mean number of defaults, as the number of firms becomes large. The model is more from regulation's point of view and its main feature is the "self- excitement". Some possible extensions will also be discussed. This is an ongoing joint work with Jaksa Cvitanic and Jin Ma.