Modelling Information Flows in Financial Markets

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What determines price changes in financial assets?

Arguably, the major cause of price changes is "new information".

When a new piece of information circulates in a financial market (whether true, or bogus), the prices of related assets move in response, and move again when the information is updated.

In this talk I discuss some of the issues involved in modelling the flow of information in financial markets, and I present some elementary models for "information" in various situations.

Some applications to the pricing of various types of financial products will be indicated.

Finally, I make some remarks about statistical arbitrage strategies, and about price formation in inhomogeneous markets.

Work based on:

1. D. C. Brody, L. P. Hughston & A. Macrina (2007) "Beyond hazard rates: a new framework for credit-risk modelling". In *Advances in Mathematical Finance, Festschrift volume in honour of Dilip Madan*. R. Elliott, M. Fu, R. Jarrow and Ju-Yi Yen, eds. (Basel: Birkhäuser).

2. D. C. Brody, L. P. Hughston & A. Macrina (2008) "Information-based asset pricing" *Int. J. Theor. Appl. Fin.* **11**, 107-142.

3. D. C. Brody, L. P. Hughston & A. Macrina (2008) "Dam rain and cumulative gain" *Proc. Roy. Soc. Lond.* A **464** 1801-1822.

4. A. Macrina (2006) *An Information-Based Framework for Asset Pricing: X-factor theory and its Applications*. PhD thesis, King's College London.

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8. L. P. Hughston & A. Macrina (2009) "Information-based trading". In preparation.

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Information-based asset pricing

When models are constructed for the pricing and risk management of complicated financial products, the price dynamics of the underlying simpler financial assets upon which the complicated products are based are often simply "assumed" (modulo some parametric freedom).

But even the more basic financial assets (shares, bonds, etc) are charaterized by potentially "complex" features, and so to make sense of their behaviour we need to consider what goes into the determination of their prices.

To build up models for the dynamics of asset prices, it seems logical therefore to proceed step by step along the following lines:

(A) Model the cash-flows as random variables
(B) Model the market filtration ("flow of information" to market)
(C) Model the pricing kernel (discounting, risk aversion, no arbitrage)
(D) Work out the resulting dynamics.

We model the financial markets with the specification of a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ on which we are going to explicitly *construct a filtration* $\{\mathcal{F}_t\}_{0 \leq t < \infty}$ representing the flow of information to market participants.

The markets we consider will, in general, be "incomplete".

That is to say, although derivatives can be priced we do not assume that they can be hedged.

Since we are going to model the filtration we say that we are working in an "information-based" framework.

Cash flow structures and market factors

We consider a financial contract that delivers a set of random cash-flows $\{D_{T_k}\}_{k=1,\ldots,n}$ on the dates $\{T_k\}_{k=1,\ldots,n}$.

Let the pricing kernel process be denoted by $\{\pi_t\}$.

At time t the value S_t of the contract that generates the cash flows $\{D_{T_k}\}_{k=1,\ldots,n}$ is then given by the following valuation formula:

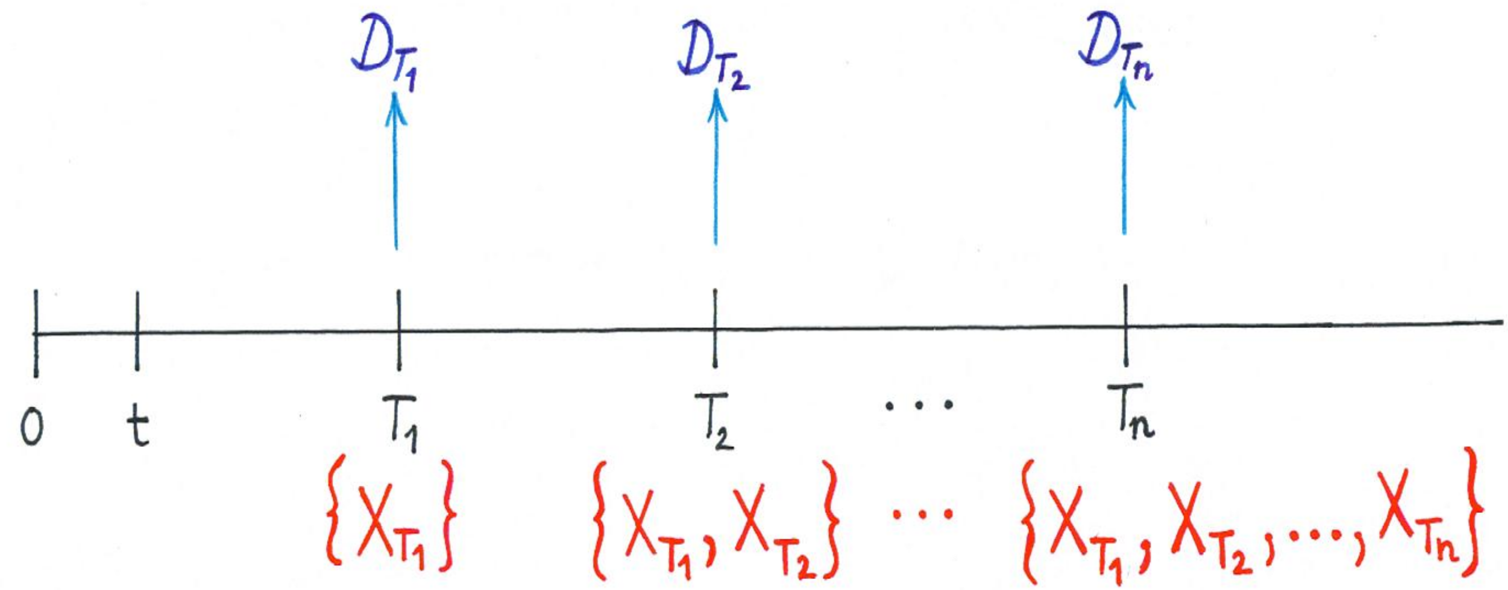
$$S_t = \frac{1}{\pi_t} \sum_{k=1}^{n} \mathbf{1}_{\{t<T_k\}} \mathbb{E}^{\mathbb{P}} \left[\pi_{T_k} D_{T_k} | \mathcal{F}_t \right] . \tag{1}$$

To make sense of this formula we need first to model the filtration, and then we need to model the pricing kernel.

To model the filtration we proceed as follows.

Let us introduce a set of independent random variables $\{X_{T_k}\}_{k=1,\ldots,n}$, which we call *market factors*.

For each k the cash flow D_{T_k} is assumed to depend on the independent market factors $X_{T_1}, X_{T_2}, \ldots, X_{T_k}$.



Thus for each date T_k we introduce a cash-flow function Δ_{T_k} such that

$$D_{T_k} = \Delta_{T_k}(X_{T_1}, X_{T_2}, \ldots, X_{T_k}). \tag{2}$$

For each asset, we need therefore to model the relevant X-factors, the associated *a priori* probabilities, and the form of the cash-flow functions.

Let us consider some simple examples.

Example I. Simple credit-risky coupon bond (two coupons, no recovery on default).

$$D_{T_1} = \mathbf{c} X_{T_1} \tag{3}$$
$$D_{T_2} = (\mathbf{c} + \mathbf{n}) X_{T_1} X_{T_2}. \tag{4}$$

Here $\mathbf{c}$ and $\mathbf{n}$ denote the coupon and principal.

X_{T_1}, X_{T_2} are independent digital variables taking the values 0 or 1 with designated *a priori* probabilities.

Example II. Credit-risky coupon bond with recovery.

$$D_{T_1} = \mathbf{c} X_{T_1} + R_1(\mathbf{c} + \mathbf{n})(1 - X_{T_1}) \tag{5}$$
$$D_{T_2} = (\mathbf{c} + \mathbf{n}) X_{T_1} X_{T_2} + R_2(\mathbf{c} + \mathbf{n}) X_{T_1}(1 - X_{T_2}). \tag{6}$$

Here R_1, R_2 are recovery rates.

Information processes

For each X-factor X_T we assume that the information available to market participants about X_T is contained in a so-called information process $\{\xi_{tT}\}_{0\leq t\leq T}$.

The information process has the property that

$$\{\xi_{TT}\} = f(X_T) \tag{7}$$

for some invertible function $f(x)$.

Thus the information process "reveals" the value of the associated X-factor at time T_k.

At earlier times, the information process contains "partial information" about the value of the X-factor. We'll come to some explicit examples of information processes shortly.

Now we are in a position to say how to construct the market filtration.

We assume that $\{\mathcal{F}_t\}$ is generated collectively by the various market information processes $\{\xi_{tT_k}\}_{k=1,\ldots,n}$.

Thus the information at time t is given by:

$$\mathcal{F}_t = \sigma\left(\left\{\xi_{sT_k}\right\}_{0\le s\le t,\ k=1,\ldots,n}\right). \tag{8}$$

Pricing kernel

Next we assume that the pricing kernel process is adapted to the $\{\mathcal{F}_t\}$.

Thus from knowledge of the history of the information processes up to time t one can work out the value of the pricing kernel at time t.

In a typical model the pricing kernel is given by the discounted marginal utility of consumption of a representative agent.

It is reasonable to suppose that the agent's consumption plan is adapted to the information filtration.

The idea is that the filtration represents the flow of information available at each time t about the X-factors, and that the agent's consumption is determined by this information. In other words, the agent behaves "rationally".

Brownian bridge information

To construct explicit models it is useful to transform to the risk neutral measure.

This can be achieved by use of the pricing kernel. We make the additional simplifying assumption in what follows that the interest rate system is deterministic.

Then the valuation formula takes the form

$$S_t = \sum_{k=1}^{n} \mathbf{1}_{\{t<T_k\}} P_{tT_k} \mathbb{E}^{\mathbb{Q}} \left[D_{T_k} | \mathcal{F}_t \right]. \tag{9}$$

Absence of arbitrage implies that the default-free discount bond system

$\{P_{tT}\}_{0\leq t\leq T<\infty}$ is of the form

$$P_{tT} = \frac{P_{0T}}{P_{0t}}, \tag{10}$$

where $\{P_{0t}\}_{0\leq t<\infty}$ is the initial term structure, which we regard as being given.

For each X-factor X_{T_k} we take the associated information process to be of the form

$$\xi_{tT_k} = \sigma_{T_k}\, t\, X_{T_k} + \beta_{tT_k}. \tag{11}$$

The process $\{\beta_{tT_k}\}$ is a standard $\mathbb{Q}$-Brownian bridge over the time interval $[0, T_k]$, with mean zero and variance $t(T_k - t)/T_k$.

The X-factors and the Brownian bridges are assumed to be $\mathbb{Q}$-independent.

Thus under $\mathbb{Q}$ the Brownian bridges represent "market noise" and only the "signal" terms $\sigma_{T_k}\, t\, X_{T_k}$ contain "true market information".

The parameter σ_{T_k} can be interpreted as the "information flow rate" for the factor X_{T_k}.

A calculation shows that the information processes $\{\xi_{tT_k}\}$ satisfy the Markov property.

This feature implies a number of simplifications in the resulting models.

Example III. Single-dividend paying asset.

Let the cash-flow function of an asset paying a single dividend be given by $D_T = X_T$, where the market factor X_T is a continuous non-negative random variable with *a priori* $\mathbb{Q}$-density $p(x)$.

The price of such an asset can be written in the form

$$\begin{aligned} S_t &= P_{tT}\mathbb{E}\left[D_T|\mathcal{F}_t\right] && (12) \\ &= P_{tT}\int_0^\infty x\, p_t(x)\,\mathrm{d}x, && (13) \end{aligned}$$

where $p_t(x)$ is the conditional probability density function for X_T.

Making use of the The Markov property of $\{\xi_{tT}\}$ together with Bayes formula we can show that $p_t(x)$ is given by

$$p_t(x) = \frac{p(x)\exp\left[\frac{T}{T-t}(\sigma x\xi_{tT} - \frac{1}{2}\sigma^2x^2t)\right]}{\int_0^\infty p(x)\exp\left[\frac{T}{T-t}(\sigma x\xi_{tT} - \frac{1}{2}\sigma^2x^2t)\right]\mathrm{d}x}. \quad (14)$$

Thus at each time t the price of the asset is determined by the random value of the information ξ_{tT} available at that time.

Dynamic asset pricing

In the case of a single-dividend-paying asset we can obtain the dynamics of the price of the asset by applying Ito's lemma to the risk-neutral valuation formula for $t < T$:

$$S_t = P_{tT}\mathbb{E}_t\left[X_T\right]. \tag{15}$$

The result is:

$$\mathrm{d}S_t = r_t S_t \mathrm{d}t + P_{tT}\frac{\sigma T}{T-t}\mathrm{Var}_t\left[X_T\right]\mathrm{d}W_t. \tag{16}$$

Here $\mathrm{Var}_t\left[X_T\right]$ denotes the conditional variance of X_T, which is given by a function of t and ξ_{tT}.

The $\{\mathcal{F}_t\}$-adapted process $\{W_t\}$ driving the dynamics of the asset is not given exogenously, but rather is defined in terms of the information process for $t < T$ by the following formula:

$$W_t = \xi_{tT} - \int_0^t \frac{1}{T-s}\left(\sigma T\mathbb{E}_s\left[X_T\right] - \xi_{sT}\right)\mathrm{d}s. \tag{17}$$

One can then verify by use of the Levy criterion that the process $\{W_t\}$ thus defined is an $\{\mathcal{F}_t\}$-Brownian motion.

Volatility and correlation

In the case of a multiple-dividend-paying asset the price is given by

$$S_t = \sum_{k=1}^{n} \mathbf{1}_{\{t<T_k\}} P_{tT_k} \mathbb{E}_t \left[D_{T_k} \right] . \tag{18}$$

Making use of the cash-flow function $D_{T_k} = \Delta_{T_k}\left(X_1, \ldots, X_{T_k}\right)$, we obtain

$$\begin{aligned} \mathrm{d}S_t \; &= \; r_t \, S_t \, \mathrm{d}t + \sum_{k=1}^{n} \Delta_{T_k} \mathrm{d}\mathbf{1}_{\{t<T_k\}} \\ &+ \; \sum_{k=1}^{n} \mathbf{1}_{\{t<T_k\}} \, P_{tT_k} \sum_{j=1}^{k} \frac{\sigma_j T_j}{T_j - t} \, \mathrm{Cov}_t \left[\Delta_{T_k}, X_{T_j} \right] \mathrm{d}W_t^j . \end{aligned} \tag{19}$$

The $\{\mathcal{F}_t\}$-adapted Brownian motions $\{W_t^j\}$ driving the asset price are given in terms of the information processes by

$$W_t^j = \xi_{tT_j} - \int_0^t \frac{1}{T_j - s} (\sigma_j \, T_j \, \mathbb{E}_s \left[X_{T_j} \right] - \xi_{sT_j}) \mathrm{d}s. \tag{20}$$

As a consequence, we see that if an asset delivers one or more cash flows depending on two or more market factors, then it will exhibit unhedgeable stochastic volatility.

A model for dynamic correlation

Assets exhibit dynamic correlation when they share one or more X-factors in common.

As an example let us consider a pair of credit-risky discount bonds.

The first bond is defined by the cash flow D_{T_1} at T_1. The second bond is defined by the cash flow D_{T_2} at time $T_2 > T_1$.

The cash flow structure is given by:

$$\begin{aligned} D_{T_1} &= \mathbf{n}_1 X_{T_1} + R_1 \mathbf{n}_1 (1 - X_{T_1}) && (21) \\ D_{T_2} &= \mathbf{n}_2 X_{T_1} X_{T_2} + R_2^a \mathbf{n}_2 (1 - X_{T_1}) X_{T_2} \\ &+ R_2^b \mathbf{n}_2 X_{T_1} (1 - X_{T_2}) + R_2^c \mathbf{n}_2 (1 - X_{T_1})(1 - X_{T_2}). && (22) \end{aligned}$$

Here, $\mathbf{n}_1$ and $\mathbf{n}_2$ denote the bond principals.

X_{T_1} and X_{T_2} are independent digital variables.

R_1, R_2^a, R_2^b, and R_2^c denote the various possible recovery rates in the case of default.

For the dynamics of the first bond price

$$S_t^{(1)} = P_{tT_1}\mathbb{E}_t\left[D_{T_1}\right] \qquad (t < T_1) \tag{23}$$

we have:

$$\mathrm{d}S_t^{(1)} = r_t S_t^{(1)}\mathrm{d}t + P_{tT_1}\frac{\sigma_1 T_1}{T_1 - t}\,\alpha\,\mathrm{Var}_t\left[X_{T_1}\right]\mathrm{d}W_t^{(1)}, \tag{24}$$

where $\alpha = \mathbf{n}_1(1 - R_1)$.

For the dynamics of the second bond price

$$S_t^{(2)} = P_{tT_2}\mathbb{E}_t\left[D_{T_2}\right] \qquad (t < T_2) \tag{25}$$

we have:

$$\begin{aligned}\mathrm{d}S_t^{(2)} &= r_t S_t^{(2)}\mathrm{d}t + P_{tT_2}\frac{\sigma_1 T_1}{T_1 - t}\left(\beta + \delta\mathbb{E}_t\left[X_{T_2}\right]\right)\mathrm{Var}_t\left[X_{T_1}\right]\mathrm{d}W_t^{(1)} \\ &\quad + P_{tT_2}\frac{\sigma_2 T_2}{T_2 - t}\left(\gamma + \delta\mathbb{E}_t\left[X_{T_1}\right]\right)\mathrm{Var}_t\left[X_{T_2}\right]\mathrm{d}W_t^{(2)},\end{aligned} \tag{26}$$

where $\beta = \mathbf{n}_2(R_2^b - R_2^c)$, $\gamma = \mathbf{n}_2(R_2^a - R_2^c)$, and $\delta = \mathbf{n}_2(1 - R_2^a - R_2^b + R_2^c)$.

The filtration $\{\mathcal{F}_s\}_{0\le s\le t}$ is generated by the information processes $\{\xi_s^{(1)}\}_{0\le s\le t}$ and $\{\xi_s^{(2)}\}_{0\le s\le t}$ associated with X_{T_1} and X_{T_2} respectively.

The dynamics of the two bonds depend on a common Brownian driver $\{W_t^{(1)}\}$.

Thus the fact that the asset payoffs share an X-factor gives rise to a dynamic correlation between the movements of the prices $\{S_t^{(1)}\}$ and $\{S_t^{(2)}\}$.The instantaneous correlation between the price movements is given by:

$$\rho_t = \frac{\mathrm{d}S_t^{(1)}\,\mathrm{d}S_t^{(2)}}{\sqrt{\left(\mathrm{d}S_t^{(1)}\right)^2\left(\mathrm{d}S_t^{(2)}\right)^2}}. \tag{27}$$

Hence, using the expressions for the respective dynamics, we obtain:

$$\rho_t = \tag{28}$$

$$\frac{\frac{\sigma_1 T_1}{T_1-t}\left(\beta+\delta\mathbb{E}_t\left[X_{T_1}\right]\right)\mathbb{V}_t\left[X_{T_1}\right]}{\sqrt{\left(\frac{\sigma_1 T_1}{T_1-t}\right)^2\left(\beta+\delta\mathbb{E}_t\left[X_{T_2}\right]\right)^2\left(\mathbb{V}_t\left[X_{T_1}\right]\right)^2+\left(\frac{\sigma_2 T_2}{T_2-t}\right)^2\left(\gamma+\delta\mathbb{E}_t\left[X_{T_1}\right]\right)^2\left(\mathbb{V}_t\left[X_{T_2}\right]\right)^2}}. \tag{29}$$

Thus we are able to calculate explicity the dynamics of the correlations between the movements of assets.

Information and statistical arbitrage

So far we have assumed that all market participants have equal access to information, but one can ask what happens if some traders are more “informed” than others.

For example, suppose we consider a financial product that pays a single cash flow X_T at time T. We can think of this product as a kind of bond.

The general market trader has access to an information process concerning X_T, but there also “informed” traders that have access (say) to two or more information processes concerning X_T.

Thus the informed trader has in some sense a “better estimate” of the true value of the bond.

Given that the informed trader is on average “more knowledgeable” than the general market participant it is natural to ask how the informed trader can take advantage of the situation, i.e. to seek out “statistical arbitrage” opportunities.

We'll assume that the informed trader operates on a relatively small scale, and that the actions of the informed trader do not significantly influence the market.

For example, suppose we consider a trading strategy such that at some designated time $t \in (0, T)$ a market trader purchases a bond iff at that time the bond price B_{tT} is greater than KP_{tT} for some specified threshold K.

The informed trader follows the same rule, but makes a better estimate for the value of the bond, and hence purchases the bond iff $\tilde{B}_{tT} > KP_{tT}$.

In either case a bond that is purchased will be held until maturity.

That such a strategy leads to a statistical arbitrage opportunity for the informed trader, but not for the general market trader, can be seen as follows.

We assume that the initial position of the trader is zero, i.e. purchase of a digital bond at t requires borrowing the amount B_{tT} at that time, and repaying the amount $P_{tT}^{-1} B_{tT}$ at T.

Thus the value of the market trader's portfolio at T is

$$V_T = \mathbb{1}\{B_{tT} > KP_{tT}\}(X_T - P_{tT}^{-1} B_{tT}), \tag{30}$$

whereas the terminal value of the informed trader's portfolio is

$$\tilde{V}_T = \mathbb{1}\{\tilde{B}_{tT} > KP_{tT}\}(X_T - P_{tT}^{-1} B_{tT}). \tag{31}$$

Consider now the present value $P_{0T}\mathbb{E}[\Delta V_T]$ of a security that delivers a cash flow equal to the excess P&L $\Delta V_T = \tilde{V}_T - V_T$ generated by the strategy of the informed trader.

By use of the tower property we have $\mathbb{E}[\Delta V_T] = \mathbb{E}[\mathbb{E}[\Delta V_T|\mathcal{G}_t]]$. But:

$$\mathbb{E}[\Delta V_T|\mathcal{G}_t] = P_{tT}^{-1}\left(\mathbb{1}\{\tilde{B}_{tT} > KP_{tT}\} - \mathbb{1}\{B_{tT} > KP_{tT}\}\right)\left(\tilde{B}_{tT} - B_{tT}\right), \quad (32)$$

since the random variables B_{tT} and $\tilde{B}_{tT}$ are both $\mathcal{G}_t$-measurable.

If $\tilde{B}_{tT} > B_{tT}$ then $\mathbb{1}\{\tilde{B}_{tT} > KP_{tT}\} - \mathbb{1}\{B_{tT} > KP_{tT}\}$ is nonnegative, whereas if $\tilde{B}_{tT} < B_{tT}$ then $\mathbb{1}\{\tilde{B}_{tT} > KP_{tT}\} - \mathbb{1}\{B_{tT} > KP_{tT}\}$ is nonpositive.

It follows that $\mathbb{E}[\Delta V_T|\mathcal{G}_t]$ is a nonnegative random variable, and hence $\mathbb{E}[\Delta V_T] > 0$, since $\mathbb{E}[\Delta V_T|\mathcal{G}_t] > 0$ with probability greater than zero.

We know that the present value of the payoff of the strategy of the general market trader must be zero.

Therefore, the informed trader can execute a transaction at zero cost that has positive value, and this is what we mean by "statistical arbitrage".

Price formation in inhomogeneous markets

Alternatively we can consider the situation where a market has a number of traders operating within it, and where all of the traders are more or less on an equal footing, but where different traders have access to different information.

Thus we consider the problem of "price formation".

Let us consider as an example a market with two traders, labelled trader 1 and trader 2.

As before, there is a single asset, with payout X at time T.

The traders have access to separate sources of information about X, given respectively by:

$$\xi_t^1 = \sigma^1 tX + \beta_{tT}^1 \tag{33}$$

and

$$\xi_t^2 = \sigma^2 tX + \beta_{tT}^2. \tag{34}$$

Here $\{\beta_{tT}^1\}$ and $\{\beta_{tT}^2\}$ are independent Brownian bridges.

Trader 1 works out the price $S_t^1 = P_{tT}\mathbb{E}[X|\xi_t^1]$ that he knows the whole market would make if they had the same information as he does.

Likewise, trader 2 works out the price $S_t^2 = P_{tT}\mathbb{E}[X|\xi_t^2]$ that she knows the whole market would make if they had the same information as she does.

Traders 1 and 2 are unaware of each other's prices, but can gain information by trading.

The trading process works as follows.

Each trader makes a spread about their price. Letting $0 < \phi^- < 1 < \phi^+$, we set

$$S_t^{1\pm} = \phi^\pm S_t^1 \tag{35}$$

for the buy price and sell price made by trader 1 at time t.

Thus trader 1 is willing to buy at a price slightly below his information-based price S_t^1, and is willing to sell at a price that is slightly above that price.

Likewise trader 2 is willing to buy at a price slightly below her information-based price S_t^2, and is willing to sell at a price that is slightly above that price:

$$S_t^{2\pm} = \phi^\pm S_t^2. \tag{36}$$

We assume that there is a "match maker" or exchange that continuously monitors the prices made by the traders on a confidential basis.

The exchange effects a trade of some fixed size when the buy price of one of the traders reaches the level of the sell price of the other trader.

That is to say, a trade takes place when

$$S_t^{1-} = S_t^{2+} \tag{37}$$

or

$$S_t^{1+} = S_t^{2-}. \tag{38}$$

But when a trade occurs, at that moment each trader learns the other's price, and as a consequence can back out the value of the corresponding information process.

Therefore when a trade occurs the traders each briefly have access to both pieces of information, and are thus in a position to make a better price, namely that given by:

$$S_t^{12} = P_{tT}\mathbb{E}[X|\xi_t^1, \xi_t^2]. \tag{39}$$

We conclude that immediately after a trade the information-based prices made by each of the traders will jump to the same level, and that the *a priori* probability distribution for X will be updated correspondingly.

Once the trade is concluded, the link between the two traders is lost, and each trader again has access only to their own information source.

Starting from the same price, the prices made by the two traders diverge as they receive different information going forward. A further trade will then occur when the buy price of one of the traders next hits the sell price of the other trader.

It is evident that by this mechanism the market price is stabilised.

The general situation, where there are a number of traders present in the market, and where the asset cash flows depend on a number of market factors, is very rich.

It is evident that in the broad picture there is no universal filtration, nor a universal pricing measure. Nevertheless by exchanging information through trading activity market participants can maintain a "law of reasonable price range" if not a "law of one price".

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