

# Appendix of Exploiting disagreement between high-dimensional variable selectors for uncertainty visualization

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## A Appendix

### A.1 Proof of Proposition 1

*Proof.*

$$\begin{aligned}
& \arg \min_M \frac{1}{|\mathcal{M}|} \sum_{\{k|M_k \in \mathcal{M}\}} \text{Hamming}(M_k, M) \\
\iff & \arg \min_M \frac{1}{|\mathcal{M}|} \sum_{\{k|M_k \in \mathcal{M}\}} \sum_{j=1}^p (\mathbb{1}_{\beta_j^k=0} \mathbb{1}_{j \in M} + \mathbb{1}_{\beta_j^k \neq 0} \mathbb{1}_{j \notin M}) \\
\iff & \arg \min_M \frac{1}{|\mathcal{M}|} \sum_{j=1}^p \sum_{\{k|M_k \in \mathcal{M}\}} [(1 - \mathbb{1}_{\beta_j^k \neq 0}) \mathbb{1}_{j \in M} + \mathbb{1}_{\beta_j^k \neq 0} (1 - \mathbb{1}_{j \in M})] \\
\iff & \arg \min_M \frac{1}{|\mathcal{M}|} \sum_{j=1}^p \sum_{\{k|M_k \in \mathcal{M}\}} [\mathbb{1}_{j \in M} (1 - 2\mathbb{1}_{\beta_j^k \neq 0})] \\
\iff & \arg \min_M \sum_{j=1}^p \mathbb{1}_{j \in M} \left(1 - 2 \frac{\sum_{\{k|M_k \in \mathcal{M}\}} \mathbb{1}_{\beta_j^k \neq 0}}{|\mathcal{M}|}\right) \\
\iff & \arg \min_M \sum_{j=1}^p \mathbb{1}_{j \in M} (1 - 2\tau_j)
\end{aligned}$$

□

## A.2 Proof of Proposition 2

*Proof.* Let the number of fitted models for covariance  $j$  to be estimated with positive sign to be  $n_j^+$  and those estimated with negative sign to be  $n_j^-$ .

$$\begin{aligned}
& \arg \min_M \frac{1}{|\mathcal{M}|} \sum_{\{k|M_k \in \mathcal{M}\}} \text{dist}(M_k, M) \\
\iff & \frac{1}{|\mathcal{M}|} \arg \min_{\{s_j^M | j=1, \dots, p\}} \sum_{\{k|M_k \in \mathcal{M}\}} |s_j^M - \text{sign}(\beta_j^k)| \\
\iff & \frac{1}{|\mathcal{M}|} \arg \min_{\{s_j^M | j=1, \dots, p\}} \sum_{\{k|M_k \in \mathcal{M}\}} [ (|\mathcal{M}| - n_j^+ + n_j^-) \mathbb{1}_{s_j^M=+1} + (n_j^+ + n_j^-) \mathbb{1}_{s_j^M=0} + (|\mathcal{M}| + n_j^+ - n_j^-) \mathbb{1}_{s_j^M=-1} ] \\
\iff & \arg \min_{\{s_j^M | j=1, \dots, p\}} \sum_{\{k|M_k \in \mathcal{M}\}} [ (1 - \tau_j^+ + \tau_j^-) \mathbb{1}_{s_j^M=+1} + (\tau_j^+ + \tau_j^-) \mathbb{1}_{s_j^M=0} + (1 - \tau_j^- + \tau_j^+) \mathbb{1}_{s_j^M=-1} ]
\end{aligned}$$

which is equivalent to setting

$$s_j^M = \begin{cases} +1 & \text{if } \tau_j^+ \geq 1/2 \\ -1 & \text{if } \tau_j^- \geq 1/2 \\ 0 & \text{otherwise} \end{cases}$$

□

## A.3 Illustration of selection disagreement via Jaccard distance heatmap

The heat maps in Figure 1-5 show the average Jaccard distance ( $\text{dist}_{\text{Jaccard}}(A, B) = \frac{|A \Delta B|}{|A \cup B|}$ , where  $A \Delta B = (A \cup B) - (A \cap B)$ ) among selected sets from five different variable selection methods under simulation setting 1-5 (see Section 5 more details). The Jaccard distance shows the relative size of selection disagreement. It ranges from 0 to 1 and larger distance means larger dissimilarity between two selected sets. The Jaccard distance among some selection methods can be quite large, showing that relatively large number of covariates are selected by one method but not by another. Even for methods that are relatively similar like the Lasso and elastic Lasso, the difference among them is still not negligible for most settings.



Figure 1: Average Jaccard distance among fitted models from different methods using simulated data from model 1

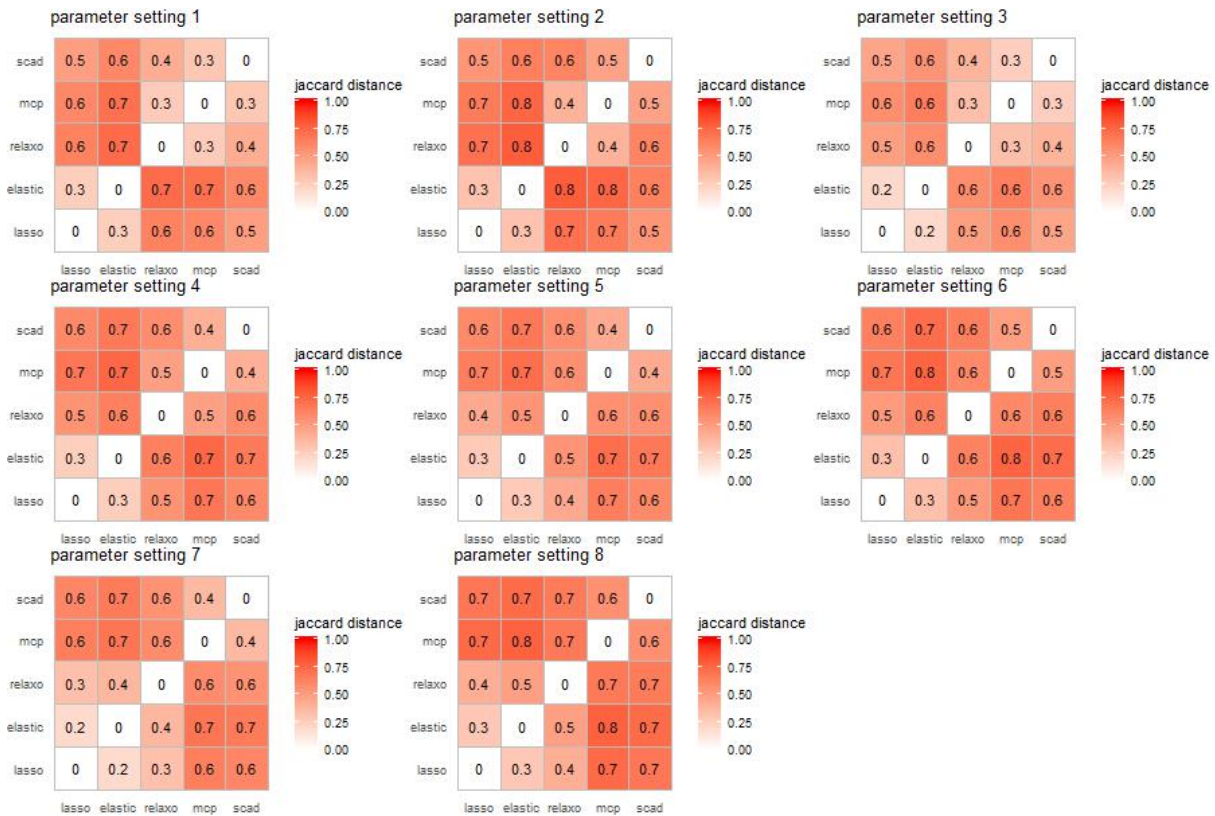


Figure 2: Average Jaccard distance among fitted models from different methods using simulated data from model 2

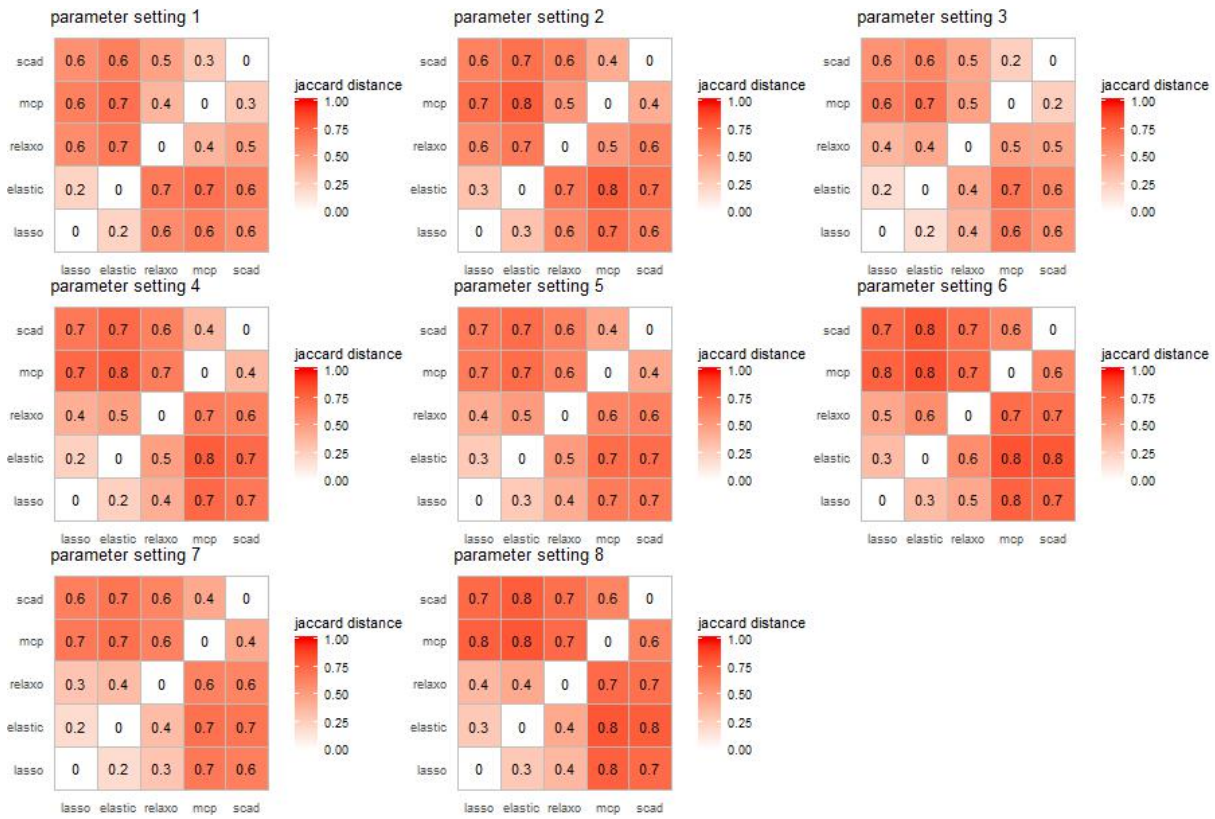


Figure 3: Average Jaccard distance among fitted models from different methods using simulated data from model 3

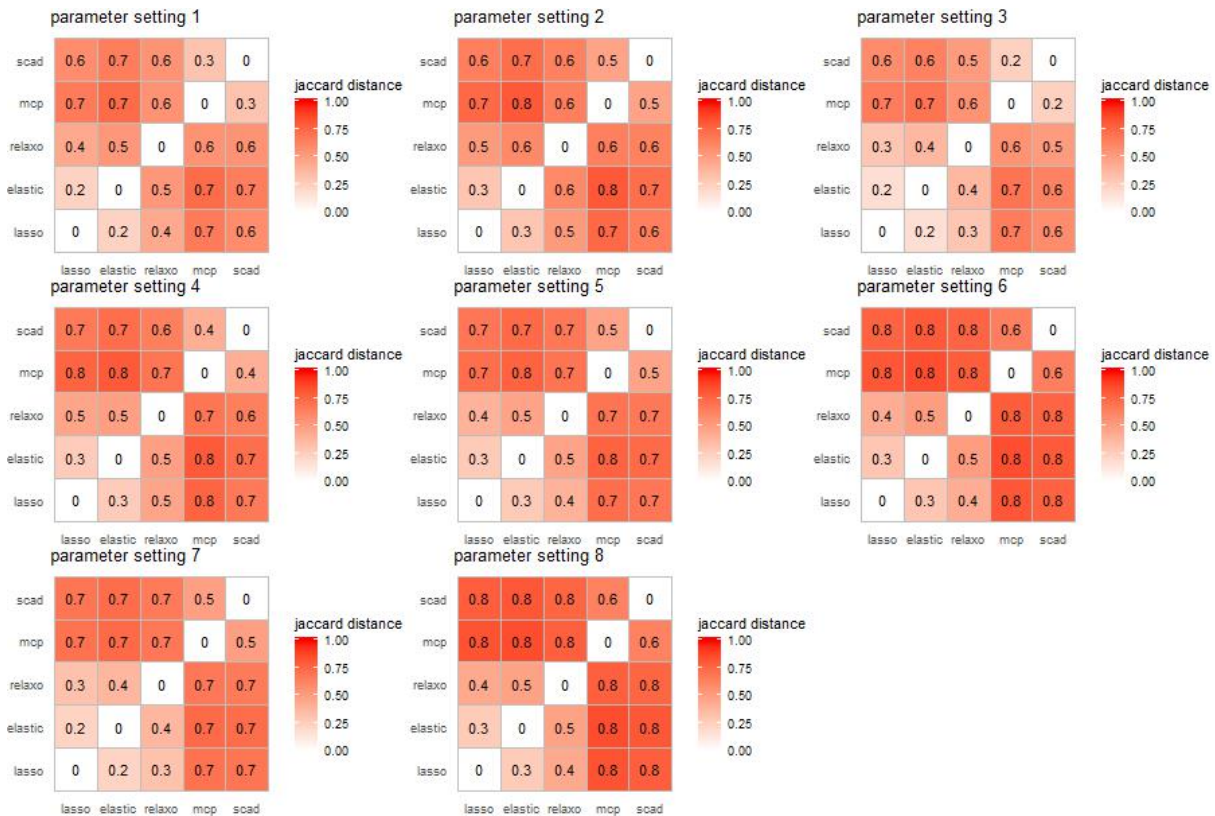


Figure 4: Average Jaccard distance among fitted models from different methods using simulated data from model 4

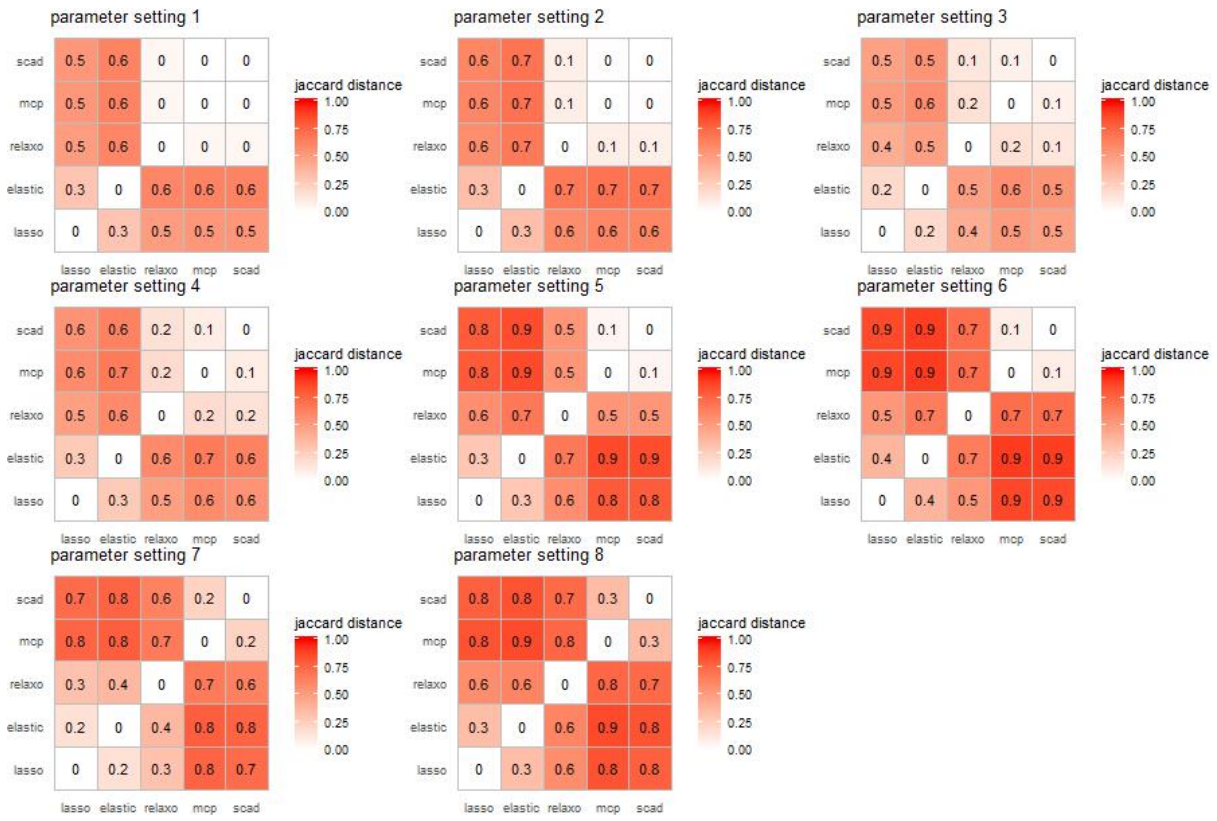


Figure 5: Average Jaccard distance among fitted models from different methods using simulated data from model 5

## A.4 Tables of the simulation results

| parameters                               | methods         | f                  | FP+FN              | FP          | FN          | pred.err            | l1.diff            | l2.diff            | size        |
|--|-----------------|--------------------|--------------------|-------------|-------------|---------------------|--------------------|--------------------|-------------|
| setting 1<br>rho = 0.5<br>s = 3<br>p = 8 | lasso           | 0.76 (0.01)        | 2.11 (0.14)        | 2.11        | 0           | 1.13 (0.02)         | 0.75 (0.04)        | 0.39 (0.02)        | 5.11 (0.14) |
|  | elastic net     | 0.71 (0.01)        | 2.67 (0.14)        | 2.67        | 0           | 1.15 (0.03)         | 0.85 (0.04)        | 0.42 (0.02)        | 5.67 (0.14) |
|  | relaxed lasso   | 0.91 (0.01)        | 0.7 (0.1)          | 0.7         | 0           | 1.11 (0.03)         | 0.6 (0.04)         | 0.34 (0.02)        | 3.7 (0.1)   |
|  | mcp             | 0.94 (0.01)        | 0.47 (0.1)         | 0.47        | 0           | 1.1 (0.02)          | 0.53 (0.04)        | 0.32 (0.02)        | 3.47 (0.1)  |
|  | scad            | 0.94 (0.01)        | 0.53 (0.11)        | 0.53        | 0           | 1.09 (0.02)         | 0.51 (0.04)        | 0.31 (0.02)        | 3.53 (0.11) |
|  | vsd             | 0.98 (0)           | 0.11 (0.03)        | 0.11        | 0           | 1.08 (0.02)         | 0.43 (0.02)        | 0.28 (0.01)        | 3.11 (0.03) |
|  | bic             | 0.96 (0.01)        | 0.35 (0.08)        | 0.35        | 0           | 1.09 (0.02)         | 0.5 (0.03)         | 0.31 (0.02)        | 3.35 (0.08) |
|  | <b>ebic</b>     | <b>0.96</b> (0.01) | 0.35 (0.08)        | 0.35        | 0           | <b>1.09</b> (0.02)  | <b>0.5</b> (0.03)  | <b>0.31</b> (0.02) | 3.35 (0.08) |
|  | <u>cv</u>       | <u>0.94</u> (0.01) | <u>0.48</u> (0.11) | <u>0.48</u> | 0           | <b>1.09</b> (0.03)  | 0.51 (0.04)        | <b>0.31</b> (0.02) | 3.48 (0.11) |
|  | <b>csuv.m.0</b> | <b>0.96</b> (0.01) | <b>0.27</b> (0.06) | <b>0.27</b> | 0           | <b>1.09</b> (0.02)  | <b>0.5</b> (0.03)  | <b>0.31</b> (0.02) | 3.27 (0.06) |
|  | <b>csuv.s.0</b> | <u>0.94</u> (0.01) | 0.42 (0.06)        | 0.42        | 0           | <u>1.1</u> (0.02)   | <u>0.55</u> (0.03) | <u>0.33</u> (0.02) | 3.42 (0.06) |
|  | csuv.m.5        | 0.96 (0.01)        | 0.27 (0.06)        | 0.27        | 0           | 1.09 (0.02)         | 0.5 (0.03)         | 0.31 (0.02)        | 3.27 (0.06) |
|  | csuv.s.5        | 0.94 (0.01)        | 0.42 (0.06)        | 0.42        | 0           | 1.1 (0.02)          | 0.55 (0.03)        | 0.33 (0.02)        | 3.42 (0.06) |
|  | csuv.m.0.all    | 0.95 (0.01)        | 0.38 (0.07)        | 0.38        | 0           | 1.1 (0.02)          | 0.53 (0.03)        | 0.33 (0.02)        | 3.38 (0.07) |
|  | csuv.s.0.all    | 0.92 (0.01)        | 0.56 (0.06)        | 0.56        | 0           | 1.11 (0.02)         | 0.59 (0.03)        | 0.35 (0.02)        | 3.56 (0.06) |
|  | csuv.m.0.mcp    | 0.97 (0.01)        | 0.21 (0.06)        | 0.21        | 0           | 1.09 (0.02)         | 0.47 (0.03)        | 0.29 (0.02)        | 3.21 (0.06) |
|  | csuv.s.0.mcp    | 0.97 (0.01)        | 0.2 (0.05)         | 0.2         | 0           | 1.08 (0.02)         | 0.46 (0.03)        | 0.29 (0.02)        | 3.2 (0.05)  |
| setting 2<br>rho = 0.5<br>s = 3<br>p = 8 | lasso           | 0.76 (0.01)        | 2.13 (0.14)        | 2.11        | 0.02        | 10.12 (0.21)        | 2.26 (0.11)        | 1.17 (0.05)        | 5.09 (0.14) |
|  | elastic net     | 0.73 (0.01)        | 2.49 (0.14)        | 2.48        | 0.01        | 10.25 (0.23)        | 2.39 (0.11)        | 1.2 (0.04)         | 5.47 (0.14) |
|  | relaxed lasso   | 0.87 (0.01)        | 1.01 (0.12)        | 0.96        | 0.05        | 10.11 (0.23)        | 1.99 (0.12)        | 1.12 (0.05)        | 3.91 (0.12) |
|  | mcp             | 0.84 (0.01)        | 1.29 (0.13)        | 1.15        | 0.14        | 10.38 (0.23)        | 2.3 (0.14)         | 1.28 (0.06)        | 4.01 (0.15) |
|  | scad            | 0.82 (0.01)        | 1.53 (0.13)        | 1.48        | 0.05        | 10.31 (0.23)        | 2.26 (0.13)        | 1.26 (0.06)        | 4.43 (0.14) |
|  | vsd             | 0.87 (0.01)        | 0.65 (0.07)        | 0.08        | 0.57        | 10.79 (0.27)        | 2.18 (0.13)        | 1.42 (0.08)        | 2.51 (0.06) |
|  | bic             | 0.86 (0.01)        | 1.04 (0.11)        | 0.92        | 0.12        | 10.24 (0.22)        | 2.11 (0.12)        | 1.22 (0.06)        | 3.8 (0.12)  |
|  | <b>ebic</b>     | 0.86 (0.01)        | 1.06 (0.11)        | 0.94        | <u>0.12</u> | <u>10.22</u> (0.22) | 2.11 (0.12)        | <u>1.22</u> (0.06) | 3.82 (0.13) |
|  | <u>cv</u>       | <u>0.76</u> (0.01) | <u>2.12</u> (0.14) | <u>2.09</u> | <b>0.03</b> | 10.13 (0.22)        | <u>2.28</u> (0.11) | 1.18 (0.05)        | 5.06 (0.14) |
|  | <b>csuv.m.0</b> | <b>0.91</b> (0.01) | <b>0.65</b> (0.09) | <b>0.56</b> | 0.09        | 10.09 (0.22)        | <b>1.83</b> (0.11) | <b>1.09</b> (0.06) | 3.47 (0.08) |
|  | <b>csuv.s.0</b> | 0.9 (0.01)         | 0.72 (0.08)        | 0.64        | 0.08        | <b>10.08</b> (0.21) | 1.88 (0.1)         | 1.11 (0.05)        | 3.56 (0.08) |
|  | csuv.m.5        | 0.91 (0.01)        | 0.65 (0.09)        | 0.56        | 0.09        | 10.09 (0.22)        | 1.83 (0.11)        | 1.09 (0.06)        | 3.47 (0.08) |
|  | csuv.s.5        | 0.9 (0.01)         | 0.72 (0.08)        | 0.64        | 0.08        | 10.08 (0.21)        | 1.88 (0.1)         | 1.11 (0.05)        | 3.56 (0.08) |
|  | csuv.m.0.all    | 0.91 (0.01)        | 0.63 (0.09)        | 0.57        | 0.06        | 10.09 (0.22)        | 1.83 (0.11)        | 1.09 (0.06)        | 3.51 (0.08) |
|  | csuv.s.0.all    | 0.9 (0.01)         | 0.73 (0.08)        | 0.67        | 0.06        | 10 (0.21)           | 1.86 (0.1)         | 1.1 (0.05)         | 3.61 (0.07) |
|  | csuv.m.0.mcp    | 0.9 (0.01)         | 0.74 (0.1)         | 0.57        | 0.17        | 10.25 (0.23)        | 1.99 (0.13)        | 1.18 (0.07)        | 3.4 (0.11)  |
|  | csuv.s.0.mcp    | 0.9 (0.01)         | 0.69 (0.09)        | 0.55        | 0.14        | 10.15 (0.22)        | 1.9 (0.12)         | 1.14 (0.06)        | 3.41 (0.08) |
| setting 3<br>rho = 0.5<br>s = 3<br>p = 8 | lasso           | 0.73 (0.02)        | 2.16 (0.14)        | 1.86        | 0.3         | 40.51 (0.85)        | 4.33 (0.21)        | 2.28 (0.09)        | 4.56 (0.16) |
|  | elastic net     | 0.71 (0.01)        | 2.39 (0.14)        | 2.14        | 0.25        | 40.66 (0.9)         | 4.37 (0.2)         | 2.25 (0.08)        | 4.89 (0.16) |
|  | relaxed lasso   | 0.73 (0.02)        | 1.78 (0.12)        | 1.13        | 0.65        | 41.41 (0.91)        | 4.52 (0.21)        | 2.51 (0.1)         | 3.48 (0.16) |
|  | mcp             | 0.66 (0.02)        | 2.26 (0.13)        | 1.41        | 0.85        | 42.76 (0.98)        | 5.54 (0.23)        | 2.97 (0.1)         | 3.56 (0.2)  |
|  | scad            | 0.7 (0.02)         | 2.22 (0.14)        | 1.67        | 0.55        | 42.47 (0.92)        | 5.51 (0.22)        | 2.94 (0.1)         | 4.12 (0.17) |
|  | vsd             | 0.54 (0.01)        | 2.24 (0.06)        | 0.01        | 2.23        | 48.66 (1.21)        | 5.94 (0.14)        | 3.53 (0.08)        | 0.78 (0.06) |
|  | bic             | 0.69 (0.02)        | 1.9 (0.12)         | 1.03        | 0.87        | 42.24 (0.95)        | 4.87 (0.21)        | 2.72 (0.1)         | 3.16 (0.16) |
|  | <b>ebic</b>     | <u>0.69</u> (0.02) | 1.9 (0.12)         | 1.03        | 0.87        | 42.24 (0.95)        | <u>4.87</u> (0.21) | <u>2.72</u> (0.1)  | 3.16 (0.16) |
|  | <u>cv</u>       | <u>0.73</u> (0.02) | <u>2.16</u> (0.14) | <u>1.86</u> | <b>0.3</b>  | <b>40.51</b> (0.85) | <b>4.33</b> (0.21) | <b>2.28</b> (0.09) | 4.56 (0.16) |
|  | <b>csuv.m.0</b> | <b>0.73</b> (0.02) | <b>1.45</b> (0.09) | <b>0.43</b> | <u>1.02</u> | <u>42.38</u> (0.91) | 4.47 (0.21)        | 2.69 (0.1)         | 2.41 (0.11) |
|  | <b>csuv.s.0</b> | <b>0.73</b> (0.02) | 1.53 (0.11)        | 0.63        | 0.9         | 42.16 (0.9)         | 4.46 (0.22)        | 2.6 (0.1)          | 2.73 (0.1)  |
|  | csuv.m.5        | 0.73 (0.02)        | 1.45 (0.09)        | 0.43        | 1.02        | 42.38 (0.91)        | 4.47 (0.21)        | 2.69 (0.1)         | 2.41 (0.11) |
|  | csuv.s.5        | 0.73 (0.02)        | 1.53 (0.11)        | 0.63        | 0.9         | 42.16 (0.9)         | 4.46 (0.22)        | 2.6 (0.1)          | 2.73 (0.1)  |
|  | csuv.m.0.all    | 0.74 (0.02)        | 1.41 (0.1)         | 0.44        | 0.97        | 42.15 (0.89)        | 4.39 (0.21)        | 2.65 (0.1)         | 2.47 (0.11) |
|  | csuv.s.0.all    | 0.74 (0.02)        | 1.51 (0.1)         | 0.65        | 0.86        | 41.99 (0.87)        | 4.4 (0.2)          | 2.6 (0.1)          | 2.79 (0.1)  |
|  | csuv.m.0.mcp    | 0.69 (0.02)        | 1.59 (0.09)        | 0.36        | 1.23        | 43.2 (0.96)         | 4.77 (0.21)        | 2.85 (0.1)         | 2.13 (0.11) |
|  | csuv.s.0.mcp    | 0.7 (0.02)         | 1.6 (0.09)         | 0.45        | 1.15        | 42.74 (0.95)        | 4.77 (0.22)        | 2.8 (0.1)          | 2.3 (0.1)   |

Table 1: Model 1: performance of CSUV and methods it compares with. Variable selection performance in terms of F-measure (f), total error (FP+FN), false positives (FP) and false negatives (FN), prediction error in terms of mse (pred.err) and estimation error in terms of l1 and l2 distance (l1.diff and l2.diff) and are shown. The numbers are based on 100 simulations. The last 8 rows are the performance of CSUV with different parameters (e.g. csuv.m.0.mcp corresponds to CSUV with MCP as constituent method and  $r = 0$ ). A bold number represents the best result among delete- $n/2$  cross validation, eBIC and CSUV using Lasso, MCP and SCAD while a underlined number represents the worst among them. Standard errors are shown inside the parentheses.

| parameters                                | methods         | f                  | FP+FN               | FP           | FN          | pred.err           | l1.diff            | l2.diff            | size         |
|---|-----------------|--------------------|---------------------|--------------|-------------|--------------------|--------------------|--------------------|--------------|
| setting 1<br>rho = 0<br>s = 5<br>p = 100  | lasso           | 0.43 (0.01)        | 15.7 (0.89)         | 15.7         | 0           | 1.28 (0.02)        | 1.76 (0.07)        | 0.53 (0.01)        | 20.7 (0.89)  |
|   | elastic net     | 0.35 (0.01)        | 21.2 (0.91)         | 21.2         | 0           | 1.36 (0.02)        | 2.22 (0.07)        | 0.6 (0.01)         | 26.2 (0.91)  |
|   | relaxed lasso   | 0.88 (0.02)        | 2.1 (0.47)          | 2.05         | 0.05        | 1.14 (0.02)        | 0.81 (0.07)        | 0.35 (0.02)        | 7 (0.47)     |
|   | mcp             | 0.85 (0.02)        | 2.3 (0.32)          | 2.3          | 0           | 1.1 (0.02)         | 0.68 (0.04)        | 0.3 (0.01)         | 7.3 (0.32)   |
|   | scad            | 0.66 (0.02)        | 6.16 (0.4)          | 6.16         | 0           | 1.09 (0.02)        | 0.75 (0.03)        | 0.3 (0.01)         | 11.16 (0.4)  |
|   | vsd             | 0.99 (0)           | 0.09 (0.03)         | 0.02         | 0.07        | 1.07 (0.02)        | 0.44 (0.02)        | 0.24 (0.01)        | 4.95 (0.03)  |
|   | bic             | 0.77 (0.02)        | 3.6 (0.33)          | 3.6          | 0           | 1.09 (0.02)        | 0.7 (0.03)         | 0.3 (0.01)         | 8.6 (0.33)   |
|   | <b>ebic</b>     | 0.85 (0.02)        | 2.31 (0.31)         | 2.31         | <b>0</b>    | 1.09 (0.02)        | 0.67 (0.04)        | 0.3 (0.01)         | 7.31 (0.31)  |
|   | <b>cv</b>       | <u>0.76</u> (0.02) | <u>4.28</u> (0.46)  | <u>4.28</u>  | <b>0</b>    | 1.09 (0.02)        | <u>0.7</u> (0.03)  | 0.29 (0.01)        | 9.28 (0.46)  |
|   | <b>csuv.m.0</b> | <b>0.98</b> (0)    | <b>0.2</b> (0.04)   | <b>0.18</b>  | <u>0.02</u> | <b>1.06</b> (0.02) | <b>0.47</b> (0.02) | <b>0.25</b> (0.01) | 5.16 (0.04)  |
|   | <b>csuv.s.0</b> | 0.93 (0.01)        | 0.81 (0.09)         | 0.79         | <u>0.02</u> | <u>1.1</u> (0.02)  | 0.62 (0.03)        | <u>0.32</u> (0.01) | 5.77 (0.09)  |
|   | csuv.m.5        | 0.99 (0)           | 0.15 (0.04)         | 0.12         | 0.03        | 1.06 (0.02)        | 0.45 (0.02)        | 0.25 (0.01)        | 5.09 (0.04)  |
|   | csuv.s.5        | 0.86 (0.01)        | 1.63 (0.1)          | 1.62         | 0.01        | 1.14 (0.02)        | 0.81 (0.03)        | 0.38 (0.01)        | 6.61 (0.1)   |
|   | csuv.m.0.all    | 0.98 (0)           | 0.21 (0.04)         | 0.19         | 0.02        | 1.06 (0.02)        | 0.47 (0.02)        | 0.25 (0.01)        | 5.17 (0.04)  |
|   | csuv.s.0.all    | 0.92 (0.01)        | 0.92 (0.09)         | 0.9          | 0.02        | 1.11 (0.02)        | 0.65 (0.03)        | 0.33 (0.01)        | 5.88 (0.09)  |
|   | csuv.m.0.mcp    | 0.99 (0)           | 0.09 (0.03)         | 0.04         | 0.05        | 1.06 (0.02)        | 0.43 (0.02)        | 0.24 (0.01)        | 4.99 (0.03)  |
| csuv.s.0.mcp                              | 0.97 (0)        | 0.29 (0.05)        | 0.28                | 0.01         | 1.07 (0.02) | 0.49 (0.02)        | 0.26 (0.01)        | 5.27 (0.05)        |              |
| setting 2<br>rho = 0<br>s = 5<br>p = 300  | lasso           | 0.33 (0.01)        | 24.65 (1.43)        | 24.64        | 0.01        | 1.43 (0.03)        | 2.41 (0.1)         | 0.65 (0.02)        | 29.63 (1.44) |
|   | elastic net     | 0.26 (0.01)        | 32.75 (1.44)        | 32.74        | 0.01        | 1.59 (0.03)        | 3.05 (0.1)         | 0.75 (0.02)        | 37.73 (1.44) |
|   | relaxed lasso   | 0.78 (0.02)        | 4.25 (0.81)         | 4.12         | 0.13        | 1.28 (0.03)        | 1.3 (0.09)         | 0.49 (0.02)        | 8.99 (0.81)  |
|   | mcp             | 0.75 (0.02)        | 4.24 (0.39)         | 4.23         | 0.01        | 1.13 (0.02)        | 0.87 (0.04)        | 0.36 (0.01)        | 9.22 (0.39)  |
|   | scad            | 0.5 (0.02)         | 12.21 (0.72)        | 12.2         | 0.01        | 1.14 (0.02)        | 1.1 (0.04)         | 0.37 (0.01)        | 17.19 (0.72) |
|   | vsd             | 0.94 (0.01)        | 0.5 (0.09)          | 0            | 0.5         | 1.28 (0.05)        | 0.76 (0.06)        | 0.43 (0.03)        | 4.5 (0.09)   |
|   | bic             | 0.55 (0.02)        | 12.38 (1.43)        | 12.37        | 0.01        | 1.19 (0.02)        | 1.32 (0.11)        | 0.41 (0.02)        | 17.36 (1.43) |
|   | <b>ebic</b>     | 0.74 (0.02)        | 5.84 (1.22)         | 5.83         | <b>0.01</b> | 1.15 (0.02)        | 0.98 (0.09)        | 0.37 (0.01)        | 10.82 (1.22) |
|   | <b>cv</b>       | <u>0.61</u> (0.02) | <u>8.92</u> (0.75)  | <u>8.91</u>  | <b>0.01</b> | 1.13 (0.02)        | <u>1.01</u> (0.05) | 0.36 (0.01)        | 13.9 (0.75)  |
|   | <b>csuv.m.0</b> | <b>0.97</b> (0.01) | <b>0.28</b> (0.06)  | <b>0.13</b>  | <u>0.15</u> | <b>1.11</b> (0.02) | <b>0.58</b> (0.03) | <b>0.31</b> (0.02) | 4.98 (0.06)  |
|   | <b>csuv.s.0</b> | 0.86 (0.01)        | 1.71 (0.12)         | 1.65         | 0.06        | <u>1.19</u> (0.02) | 0.95 (0.04)        | <u>0.44</u> (0.01) | 6.59 (0.13)  |
|   | csuv.m.5        | 0.97 (0.01)        | 0.3 (0.05)          | 0.11         | 0.19        | 1.12 (0.02)        | 0.6 (0.04)         | 0.33 (0.02)        | 4.92 (0.05)  |
|   | csuv.s.5        | 0.78 (0.01)        | 3.02 (0.15)         | 3            | 0.02        | 1.25 (0.02)        | 1.24 (0.04)        | 0.51 (0.01)        | 7.98 (0.15)  |
|   | csuv.m.0.all    | 0.97 (0.01)        | 0.31 (0.06)         | 0.15         | 0.16        | 1.12 (0.02)        | 0.59 (0.04)        | 0.32 (0.02)        | 4.99 (0.05)  |
|   | csuv.s.0.all    | 0.85 (0.01)        | 1.87 (0.14)         | 1.81         | 0.06        | 1.19 (0.02)        | 1 (0.04)           | 0.46 (0.01)        | 6.75 (0.14)  |
|   | csuv.m.0.mcp    | 0.97 (0.01)        | 0.24 (0.05)         | 0.01         | 0.23        | 1.13 (0.03)        | 0.58 (0.03)        | 0.33 (0.02)        | 4.78 (0.05)  |
| csuv.s.0.mcp                              | 0.92 (0.01)     | 0.83 (0.07)        | 0.72                | 0.11         | 1.15 (0.02) | 0.74 (0.03)        | 0.38 (0.01)        | 5.61 (0.09)        |              |
| setting 3<br>rho = 0<br>s = 10<br>p = 100 | lasso           | 0.46 (0.01)        | 24.88 (0.93)        | 24.88        | 0           | 1.55 (0.03)        | 3.23 (0.1)         | 0.73 (0.01)        | 34.88 (0.93) |
|   | elastic net     | 0.42 (0.01)        | 29.6 (0.89)         | 29.6         | 0           | 1.65 (0.03)        | 3.74 (0.1)         | 0.79 (0.01)        | 39.6 (0.89)  |
|   | relaxed lasso   | 0.8 (0.01)         | 5.53 (0.46)         | 5.49         | 0.04        | 1.36 (0.03)        | 1.95 (0.08)        | 0.58 (0.02)        | 15.45 (0.46) |
|   | mcp             | 0.88 (0.01)        | 3.1 (0.31)          | 3.1          | 0           | 1.19 (0.02)        | 1.23 (0.05)        | 0.42 (0.01)        | 13.1 (0.31)  |
|   | scad            | 0.7 (0.01)         | 8.98 (0.41)         | 8.98         | 0           | 1.2 (0.02)         | 1.4 (0.05)         | 0.43 (0.01)        | 18.98 (0.41) |
|   | vsd             | 0.98 (0.01)        | 0.4 (0.09)          | 0.1          | 0.3         | 1.25 (0.05)        | 1.06 (0.06)        | 0.43 (0.03)        | 9.8 (0.08)   |
|   | bic             | 0.83 (0.01)        | 5.28 (0.72)         | 5.28         | 0           | 1.22 (0.03)        | 1.41 (0.1)         | 0.44 (0.02)        | 15.28 (0.72) |
|   | <b>ebic</b>     | 0.87 (0.01)        | 3.27 (0.33)         | 3.27         | <b>0</b>    | <b>1.19</b> (0.02) | 1.24 (0.05)        | <b>0.42</b> (0.01) | 13.27 (0.33) |
|   | <b>cv</b>       | <u>0.63</u> (0.02) | <u>14.87</u> (1.16) | <u>14.87</u> | <b>0</b>    | <u>1.38</u> (0.03) | <u>2.25</u> (0.14) | <u>0.58</u> (0.02) | 24.87 (1.16) |
|   | <b>csuv.m.0</b> | <b>0.97</b> (0)    | <b>0.49</b> (0.08)  | <b>0.27</b>  | <u>0.22</u> | 1.23 (0.04)        | <b>1.07</b> (0.05) | 0.43 (0.02)        | 10.05 (0.09) |
|   | <b>csuv.s.0</b> | 0.87 (0.01)        | 3.19 (0.16)         | 3.16         | 0.03        | 1.31 (0.03)        | 1.64 (0.05)        | 0.54 (0.01)        | 13.13 (0.16) |
|   | csuv.m.5        | 0.98 (0)           | 0.46 (0.08)         | 0.25         | 0.21        | 1.22 (0.04)        | 1.06 (0.05)        | 0.42 (0.02)        | 10.04 (0.08) |
|   | csuv.s.5        | 0.83 (0)           | 4.29 (0.15)         | 4.27         | 0.02        | 1.35 (0.03)        | 1.86 (0.05)        | 0.58 (0.01)        | 14.25 (0.15) |
|   | csuv.m.0.all    | 0.98 (0)           | 0.47 (0.07)         | 0.3          | 0.17        | 1.21 (0.03)        | 1.05 (0.04)        | 0.42 (0.02)        | 10.13 (0.08) |
|   | csuv.s.0.all    | 0.86 (0.01)        | 3.47 (0.16)         | 3.44         | 0.03        | 1.32 (0.03)        | 1.7 (0.05)         | 0.55 (0.01)        | 13.41 (0.17) |
|   | csuv.m.0.mcp    | 0.98 (0)           | 0.36 (0.07)         | 0.03         | 0.33        | 1.25 (0.04)        | 1.05 (0.05)        | 0.43 (0.02)        | 9.7 (0.07)   |
| csuv.s.0.mcp                              | 0.94 (0)        | 1.26 (0.08)        | 1.19                | 0.07         | 1.24 (0.04) | 1.25 (0.04)        | 0.47 (0.02)        | 11.12 (0.08)       |              |

Table 2: Model 2: performance of CSUV and methods it compares with. Variable selection performance in terms of F-measure (f), total error (FP+FN), false positives (FP) and false negatives (FN), prediction error in terms of mse (pred.err) and estimation error in terms of l1 and l2 distance (l1.diff and l2.diff) and are shown. The numbers are based on 100 simulations. The last 8 rows are the performance of CSUV with different parameters (e.g. csuv.m.0.mcp corresponds to CSUV with MCP as constituent method and  $r = 0$ ). A bold number represents the best result among delete- $n/2$  cross validation, eBIC and CSUV using Lasso, MCP and SCAD while a underlined number represents the worst among them. Standard errors are shown inside the parentheses.



| parameters                                 | methods         | f                  | FP+FN               | FP           | FN          | pred.err           | l1.diff            | l2.diff            | size         |
|--|-----------------|--------------------|---------------------|--------------|-------------|--------------------|--------------------|--------------------|--------------|
| setting 4<br>rho = 0<br>s = 10<br>p = 300  | lasso           | 0.38 (0.01)        | 36.64 (1.56)        | 36.6         | 0.04        | 1.88 (0.05)        | 4.49 (0.14)        | 0.94 (0.02)        | 46.56 (1.56) |
|  | elastic net     | 0.31 (0.01)        | 47.38 (1.54)        | 47.34        | 0.04        | 2.15 (0.05)        | 5.62 (0.14)        | 1.08 (0.02)        | 57.3 (1.55)  |
|  | relaxed lasso   | 0.66 (0.01)        | 11.84 (0.9)         | 11.61        | 0.23        | 1.7 (0.05)         | 3.2 (0.13)         | 0.81 (0.02)        | 21.38 (0.93) |
|  | mcp             | 0.81 (0.01)        | 5.19 (0.35)         | 5.16         | 0.03        | 1.24 (0.03)        | 1.46 (0.06)        | 0.48 (0.02)        | 15.13 (0.35) |
|  | scad            | 0.57 (0.01)        | 15.88 (0.53)        | 15.86        | 0.02        | 1.23 (0.03)        | 1.74 (0.06)        | 0.48 (0.02)        | 25.84 (0.53) |
|  | vsd             | 0.88 (0.02)        | 2.08 (0.28)         | 0.03         | 2.05        | 2.36 (0.24)        | 2.33 (0.22)        | 0.9 (0.08)         | 7.98 (0.29)  |
|  | bic             | 0.6 (0.02)         | 19.21 (2)           | 19.19        | 0.02        | 1.39 (0.05)        | 2.44 (0.2)         | 0.58 (0.03)        | 29.17 (2.01) |
|  | <b>ebic</b>     | 0.78 (0.02)        | 8.05 (1.42)         | 8.02         | <b>0.03</b> | <b>1.28</b> (0.03) | <b>1.75</b> (0.15) | <b>0.51</b> (0.02) | 17.99 (1.42) |
|  | <b>cv</b>       | <u>0.41</u> (0.01) | <u>33.78</u> (1.67) | <u>33.74</u> | 0.04        | <u>1.8</u> (0.05)  | <u>4.18</u> (0.16) | <u>0.88</u> (0.02) | 43.7 (1.67)  |
|  | <b>csuv.m.0</b> | <b>0.91</b> (0.01) | <b>1.51</b> (0.15)  | <b>0.1</b>   | <u>1.41</u> | 1.77 (0.08)        | 1.84 (0.1)         | 0.8 (0.04)         | 8.69 (0.13)  |
|  | <b>csuv.s.0</b> | 0.76 (0.01)        | 6.53 (0.25)         | 6.29         | 0.24        | 1.5 (0.04)         | 2.35 (0.07)        | 0.7 (0.02)         | 16.05 (0.27) |
|  | csuv.m.5        | 0.91 (0.01)        | 1.59 (0.15)         | 0.08         | 1.51        | 1.82 (0.08)        | 1.89 (0.11)        | 0.82 (0.04)        | 8.57 (0.14)  |
|  | csuv.s.5        | 0.72 (0.01)        | 7.75 (0.23)         | 7.51         | 0.24        | 1.55 (0.04)        | 2.57 (0.07)        | 0.73 (0.02)        | 17.27 (0.26) |
|  | csuv.m.0.all    | 0.92 (0.01)        | 1.48 (0.15)         | 0.14         | 1.34        | 1.75 (0.08)        | 1.81 (0.11)        | 0.79 (0.04)        | 8.8 (0.12)   |
|  | csuv.s.0.all    | 0.74 (0.01)        | 7.26 (0.27)         | 7.03         | 0.23        | 1.53 (0.04)        | 2.47 (0.07)        | 0.72 (0.02)        | 16.8 (0.29)  |
|  | csuv.m.0.mcp    | 0.78 (0.01)        | 3.37 (0.18)         | 0.01         | 3.36        | 2.86 (0.13)        | 3.21 (0.14)        | 1.31 (0.04)        | 6.65 (0.18)  |
| csuv.s.0.mcp                               | 0.91 (0.01)     | 1.65 (0.14)        | 0.45                | 1.2          | 1.73 (0.09) | 1.84 (0.11)        | 0.77 (0.04)        | 9.25 (0.14)        |              |
| setting 5<br>rho = 0.9<br>s = 5<br>p = 100 | lasso           | 0.39 (0.01)        | 16.4 (0.67)         | 16.19        | 0.21        | 1.28 (0.02)        | 3.51 (0.11)        | 1.04 (0.03)        | 20.98 (0.68) |
|  | elastic net     | 0.33 (0.01)        | 21.14 (0.69)        | 20.99        | 0.15        | 1.31 (0.02)        | 4.21 (0.11)        | 1.15 (0.02)        | 25.84 (0.69) |
|  | relaxed lasso   | 0.49 (0.01)        | 10.58 (0.65)        | 10           | 0.58        | 1.31 (0.02)        | 3.31 (0.13)        | 1.07 (0.03)        | 14.42 (0.7)  |
|  | mcp             | 0.67 (0.02)        | 4.26 (0.31)         | 3.17         | 1.09        | 1.29 (0.03)        | 2.48 (0.17)        | 1.09 (0.06)        | 7.08 (0.24)  |
|  | scad            | 0.65 (0.02)        | 4.91 (0.28)         | 4.06         | 0.85        | 1.24 (0.03)        | 2.01 (0.14)        | 0.91 (0.06)        | 8.21 (0.22)  |
|  | vsd             | 0.76 (0.02)        | 2.1 (0.16)          | 0.5          | 1.6         | 1.38 (0.03)        | 1.93 (0.13)        | 1.01 (0.05)        | 3.9 (0.08)   |
|  | bic             | 0.69 (0.02)        | 4.15 (0.32)         | 3.26         | 0.89        | 1.26 (0.03)        | 2.22 (0.17)        | 0.96 (0.06)        | 7.37 (0.24)  |
|  | <b>ebic</b>     | 0.71 (0.02)        | 3.67 (0.27)         | 2.77         | 0.9         | <b>1.24</b> (0.03) | 2.07 (0.15)        | 0.94 (0.06)        | 6.87 (0.2)   |
|  | <b>cv</b>       | <u>0.47</u> (0.02) | <u>13.01</u> (0.75) | <u>12.64</u> | <b>0.37</b> | 1.27 (0.02)        | <u>3.14</u> (0.14) | <u>1.01</u> (0.04) | 17.27 (0.79) |
|  | <b>csuv.m.0</b> | <b>0.77</b> (0.02) | <b>2.25</b> (0.17)  | <b>0.97</b>  | <u>1.28</u> | <u>1.33</u> (0.03) | <b>1.9</b> (0.11)  | <b>0.93</b> (0.05) | 4.69 (0.11)  |
|  | <b>csuv.s.0</b> | 0.68 (0.01)        | 4.21 (0.18)         | 3.5          | 0.71        | 1.26 (0.02)        | 2.31 (0.1)         | 0.95 (0.04)        | 7.79 (0.18)  |
|  | csuv.m.5        | 0.77 (0.02)        | 2.16 (0.16)         | 0.8          | 1.36        | 1.35 (0.03)        | 1.88 (0.11)        | 0.95 (0.04)        | 4.44 (0.11)  |
|  | csuv.s.5        | 0.66 (0.01)        | 4.45 (0.17)         | 3.77         | 0.68        | 1.26 (0.02)        | 2.37 (0.1)         | 0.95 (0.03)        | 8.09 (0.17)  |
|  | csuv.m.0.all    | 0.76 (0.02)        | 2.43 (0.17)         | 1.23         | 1.2         | 1.31 (0.03)        | 1.96 (0.11)        | 0.94 (0.04)        | 5.03 (0.14)  |
|  | csuv.s.0.all    | 0.65 (0.01)        | 4.94 (0.21)         | 4.28         | 0.66        | 1.26 (0.03)        | 2.48 (0.1)         | 0.97 (0.03)        | 8.62 (0.22)  |
|  | csuv.m.0.mcp    | 0.69 (0.02)        | 2.53 (0.14)         | 0.36         | 2.17        | 1.87 (0.07)        | 2.37 (0.12)        | 1.24 (0.05)        | 3.19 (0.09)  |
| csuv.s.0.mcp                               | 0.72 (0.02)     | 2.88 (0.18)        | 1.55                | 1.33         | 1.34 (0.03) | 2.24 (0.14)        | 1.03 (0.05)        | 5.22 (0.08)        |              |
| setting 6<br>rho = 0.9<br>s = 5<br>p = 300 | lasso           | 0.3 (0.01)         | 24.79 (1.08)        | 24.6         | 0.19        | 1.42 (0.03)        | 3.81 (0.15)        | 1.04 (0.03)        | 29.41 (1.09) |
|  | elastic net     | 0.23 (0.01)        | 34.73 (1.22)        | 34.6         | 0.13        | 1.51 (0.03)        | 4.91 (0.14)        | 1.21 (0.02)        | 39.47 (1.21) |
|  | relaxed lasso   | 0.51 (0.02)        | 10.84 (0.88)        | 10.37        | 0.47        | 1.35 (0.03)        | 3.13 (0.15)        | 0.99 (0.03)        | 14.9 (0.9)   |
|  | mcp             | 0.59 (0.02)        | 5.52 (0.33)         | 4.24         | 1.28        | 1.33 (0.03)        | 2.66 (0.16)        | 1.16 (0.06)        | 7.96 (0.24)  |
|  | scad            | 0.57 (0.02)        | 7.54 (0.42)         | 6.94         | 0.6         | 1.21 (0.02)        | 1.78 (0.12)        | 0.79 (0.05)        | 11.34 (0.36) |
|  | vsd             | 0.75 (0.02)        | 2.47 (0.2)          | 0.58         | 1.89        | 1.87 (0.1)         | 2.28 (0.16)        | 1.12 (0.07)        | 3.69 (0.19)  |
|  | bic             | 0.59 (0.02)        | 8.1 (1.1)           | 7.33         | 0.77        | 1.27 (0.03)        | 2.3 (0.19)         | 0.92 (0.05)        | 11.56 (1.12) |
|  | <b>ebic</b>     | 0.63 (0.02)        | 5.69 (0.73)         | 4.65         | 1.04        | 1.29 (0.03)        | 2.32 (0.17)        | <u>1</u> (0.06)    | 8.61 (0.72)  |
|  | <b>cv</b>       | <u>0.44</u> (0.02) | <u>16.75</u> (1.32) | <u>16.32</u> | 0.43        | 1.33 (0.03)        | <u>2.89</u> (0.19) | 0.93 (0.04)        | 20.89 (1.34) |
|  | <b>csuv.m.0</b> | <b>0.79</b> (0.02) | <b>2.01</b> (0.16)  | <b>0.91</b>  | <u>1.1</u>  | <u>1.4</u> (0.04)  | <b>1.81</b> (0.12) | 0.92 (0.05)        | 4.81 (0.1)   |
|  | <b>csuv.s.0</b> | 0.64 (0.01)        | 5.53 (0.24)         | 5.19         | <b>0.34</b> | <b>1.24</b> (0.02) | 2.31 (0.1)         | <b>0.87</b> (0.03) | 9.85 (0.21)  |
|  | csuv.m.5        | 0.8 (0.02)         | 1.87 (0.15)         | 0.76         | 1.11        | 1.43 (0.04)        | 1.75 (0.11)        | 0.9 (0.05)         | 4.65 (0.1)   |
|  | csuv.s.5        | 0.61 (0.01)        | 6.14 (0.19)         | 5.84         | 0.3         | 1.25 (0.02)        | 2.43 (0.09)        | 0.88 (0.03)        | 10.54 (0.16) |
|  | csuv.m.0.all    | 0.8 (0.02)         | 2 (0.16)            | 1            | 1           | 1.38 (0.04)        | 1.74 (0.11)        | 0.87 (0.05)        | 5 (0.11)     |
|  | csuv.s.0.all    | 0.61 (0.01)        | 6.31 (0.27)         | 5.98         | 0.33        | 1.26 (0.03)        | 2.49 (0.11)        | 0.89 (0.03)        | 10.65 (0.25) |
|  | csuv.m.0.mcp    | 0.68 (0.02)        | 2.56 (0.15)         | 0.39         | 2.17        | 2.11 (0.07)        | 2.46 (0.12)        | 1.25 (0.05)        | 3.22 (0.11)  |
| csuv.s.0.mcp                               | 0.73 (0.01)     | 2.98 (0.17)        | 2.05                | 0.93         | 1.3 (0.03)  | 2.13 (0.13)        | 0.97 (0.05)        | 6.12 (0.07)        |              |

Table 3: Model 2: performance of CSUV and methods it compares with (continue). Variable selection performance in terms of F-measure (f), total error (FP+FN), false positives (FP) and false negatives (FN), prediction error in terms of mse (pred.err) and estimation error in terms of l1 and l2 distance (l1.diff and l2.diff) and are shown. The numbers are based on 100 simulations. The last 8 rows are the performance of CSUV with different parameters (e.g. csuv.m.0.mcp corresponds to CSUV with MCP as constituent method and  $r = 0$ ). A bold number represents the best result among delete- $n/2$  cross validation, eBIC and CSUV using Lasso, MCP and SCAD while a underlined number represents the worst among them. Standard errors are shown inside the parentheses.

| parameters                                  | methods                                     | f                  | FP+FN               | FP           | FN          | pred.err           | l1.diff            | l2.diff            | size         |
|---|---|--------------------|---------------------|--------------|-------------|--------------------|--------------------|--------------------|--------------|
| setting 7<br>rho = 0.9<br>s = 10<br>p = 100 | lasso                                       | 0.47 (0.01)        | 22.4 (0.7)          | 21.74        | 0.66        | 1.49 (0.03)        | 6.3 (0.17)         | 1.49 (0.03)        | 31.08 (0.73) |
|   | elastic net                                 | 0.42 (0.01)        | 27.11 (0.73)        | 26.68        | 0.43        | 1.54 (0.03)        | 7.11 (0.17)        | 1.57 (0.03)        | 36.25 (0.73) |
|   | relaxed lasso                               | 0.54 (0.01)        | 17.19 (0.83)        | 16.23        | 0.96        | 1.53 (0.03)        | 6.31 (0.22)        | 1.55 (0.03)        | 25.27 (0.91) |
|   | mcp   | 0.64 (0.01)        | 7.8 (0.32)          | 4.56         | 3.24        | 1.66 (0.05)        | 6.15 (0.23)        | 1.98 (0.06)        | 11.32 (0.27) |
|   | scad  | 0.66 (0.01)        | 8.03 (0.35)         | 5.54         | 2.49        | 1.48 (0.04)        | 4.97 (0.21)        | 1.71 (0.06)        | 13.05 (0.3)  |
|   | vsd   | 0.7 (0.02)         | 5.13 (0.24)         | 1.36         | 3.77        | 1.86 (0.08)        | 5.3 (0.22)         | 1.89 (0.06)        | 7.59 (0.15)  |
|   | bic   | 0.67 (0.01)        | 7.44 (0.33)         | 4.84         | 2.6         | 1.55 (0.04)        | 5.35 (0.22)        | 1.78 (0.05)        | 12.24 (0.29) |
|   | <b>ebic</b>                                 | 0.68 (0.01)        | 6.73 (0.29)         | 3.94         | <u>2.79</u> | 1.55 (0.04)        | 5.21 (0.21)        | <u>1.77</u> (0.06) | 11.15 (0.23) |
|   | <b>cv</b>                                   | <u>0.47</u> (0.01) | <u>22.4</u> (0.7)   | <u>21.74</u> | <b>0.66</b> | 1.49 (0.03)        | <u>6.3</u> (0.17)  | 1.49 (0.03)        | 31.08 (0.73) |
|   | <b>csuv.m.0</b>                             | <b>0.76</b> (0.01) | <b>4.57</b> (0.2)   | <b>1.84</b>  | 2.73        | <u>1.66</u> (0.04) | <b>4.35</b> (0.14) | 1.53 (0.04)        | 9.11 (0.15)  |
|   | <b>csuv.s.0</b>                             | 0.67 (0.01)        | 8.65 (0.24)         | 7.19         | 1.46        | <b>1.45</b> (0.03) | 4.87 (0.13)        | <b>1.44</b> (0.03) | 15.73 (0.23) |
|   | csuv.m.5                                    | 0.76 (0.01)        | 4.54 (0.19)         | 1.74         | 2.8         | 1.69 (0.04)        | 4.37 (0.14)        | 1.54 (0.04)        | 8.94 (0.15)  |
|   | csuv.s.5                                    | 0.66 (0.01)        | 9 (0.23)            | 7.54         | 1.46        | 1.45 (0.03)        | 4.96 (0.12)        | 1.45 (0.03)        | 16.08 (0.22) |
|   | csuv.m.0.all                                | 0.76 (0.01)        | 4.76 (0.22)         | 2.31         | 2.45        | 1.6 (0.04)         | 4.25 (0.15)        | 1.46 (0.04)        | 9.86 (0.16)  |
|   | csuv.s.0.all                                | 0.64 (0.01)        | 9.97 (0.25)         | 8.62         | 1.35        | 1.45 (0.03)        | 5.14 (0.11)        | 1.46 (0.03)        | 17.27 (0.27) |
|   | csuv.m.0.mcp                                | 0.59 (0.01)        | 5.95 (0.18)         | 0.47         | 5.48        | 3.22 (0.11)        | 6.16 (0.18)        | 2.19 (0.05)        | 4.99 (0.12)  |
|   | csuv.s.0.mcp                                | 0.72 (0.01)        | 5.48 (0.21)         | 2.39         | 3.09        | 1.74 (0.04)        | 5.04 (0.17)        | 1.74 (0.04)        | 9.3 (0.09)   |
|   | setting 8<br>rho = 0.9<br>s = 10<br>p = 300 | lasso              | 0.39 (0.01)         | 32.18 (1.08) | 31.63       | 0.55               | 1.72 (0.04)        | 6.9 (0.19)         | 1.5 (0.03)   |
| elastic net                                 |   | 0.32 (0.01)        | 41.56 (0.99)        | 41.13        | 0.43        | 1.83 (0.04)        | 8.36 (0.16)        | 1.68 (0.03)        | 50.7 (0.97)  |
| relaxed lasso                               |   | 0.51 (0.01)        | 18.39 (0.75)        | 17.43        | 0.96        | 1.64 (0.04)        | 6.25 (0.18)        | 1.5 (0.03)         | 26.47 (0.79) |
| mcp   |   | 0.53 (0.02)        | 11.72 (0.57)        | 7.88         | 3.84        | 2.09 (0.07)        | 7.45 (0.36)        | 2.2 (0.08)         | 14.04 (0.3)  |
| scad  |   | 0.54 (0.01)        | 14.05 (0.51)        | 11.82        | 2.23        | 1.71 (0.06)        | 5.27 (0.26)        | 1.7 (0.07)         | 19.59 (0.35) |
| vsd   |   | 0.6 (0.02)         | 6.71 (0.33)         | 1.82         | 4.89        | 3.24 (0.23)        | 6.5 (0.31)         | 2.21 (0.09)        | 6.93 (0.25)  |
| bic   |   | 0.55 (0.02)        | 13.84 (1.12)        | 11.25        | 2.59        | 1.79 (0.06)        | 5.95 (0.31)        | 1.81 (0.07)        | 18.66 (1.15) |
| <b>ebic</b>                                 |   | 0.57 (0.02)        | 10.93 (0.5)         | 7.81         | 3.12        | 1.86 (0.05)        | 6.22 (0.3)         | <u>1.94</u> (0.07) | 14.69 (0.36) |
| <b>cv</b>                                   |   | <u>0.39</u> (0.01) | <u>32.18</u> (1.08) | <u>31.63</u> | <b>0.55</b> | 1.72 (0.04)        | <u>6.9</u> (0.19)  | <b>1.5</b> (0.03)  | 41.08 (1.06) |
| <b>csuv.m.0</b>                             |   | <b>0.69</b> (0.01) | <b>5.8</b> (0.23)   | <b>2.27</b>  | <u>3.53</u> | <u>2.37</u> (0.08) | <b>5.28</b> (0.19) | 1.82 (0.05)        | 8.74 (0.16)  |
| <b>csuv.s.0</b>                             |   | 0.55 (0.01)        | 14.1 (0.32)         | 12.73        | 1.37        | <b>1.65</b> (0.04) | 5.87 (0.16)        | 1.52 (0.03)        | 21.36 (0.34) |
| csuv.m.5                                    |   | 0.69 (0.01)        | 5.68 (0.22)         | 1.96         | 3.72        | 2.57 (0.1)         | 5.25 (0.17)        | 1.83 (0.05)        | 8.24 (0.16)  |
| csuv.s.5                                    |   | 0.55 (0.01)        | 14.24 (0.33)        | 12.87        | 1.37        | 1.64 (0.04)        | 5.92 (0.16)        | 1.54 (0.04)        | 21.5 (0.34)  |
| csuv.m.0.all                                |   | 0.69 (0.01)        | 5.99 (0.24)         | 2.68         | 3.31        | 2.26 (0.08)        | 5.17 (0.18)        | 1.76 (0.05)        | 9.37 (0.17)  |
| csuv.s.0.all                                |   | 0.53 (0.01)        | 15.93 (0.35)        | 14.69        | 1.24        | 1.64 (0.04)        | 6.12 (0.16)        | 1.53 (0.03)        | 23.45 (0.36) |
| csuv.m.0.mcp                                |   | 0.38 (0.01)        | 7.95 (0.16)         | 0.32         | 7.63        | 7.18 (0.21)        | 7.43 (0.16)        | 2.53 (0.04)        | 2.69 (0.13)  |
| csuv.s.0.mcp                                |   | 0.59 (0.01)        | 8.07 (0.26)         | 3.9          | 4.17        | 2.53 (0.08)        | 6.56 (0.21)        | 2.08 (0.05)        | 9.73 (0.11)  |

Table 4: Model 2: performance of CSUV and methods it compares with (continue). Variable selection performance in terms of F-measure (f), total error (FP+FN), false positives (FP) and false negatives (FN), prediction error in terms of mse (pred.err) and estimation error in terms of l1 and l2 distance (l1.diff and l2.diff) and are shown. The numbers are based on 100 simulations. The last 8 rows are the performance of CSUV with different parameters (e.g. csuv.m.0.mcp corresponds to CSUV with MCP as constituent method and  $r = 0$ ). A bold number represents the best result among delete- $n/2$  cross validation, eBIC and CSUV using Lasso, MCP and SCAD while a underlined number represents the worst among them. Standard errors are shown inside the parentheses.

| parameters  | methods         | f                  | FP+FN               | FP           | FN          | pred.err           | l1.diff            | l2.diff            | size         |
|---|-----------------|--------------------|---------------------|--------------|-------------|--------------------|--------------------|--------------------|--------------|
| setting 1<br>block.cor = 0.5<br>s = 5<br>p = 100  | lasso           | 0.38 (0.01)        | 17.85 (0.75)        | 17.84        | 0.01        | 1.31 (0.02)        | 2.35 (0.07)        | 0.68 (0.01)        | 22.83 (0.75) |
|   | elastic net     | 0.32 (0.01)        | 23.17 (0.81)        | 23.17        | 0           | 1.38 (0.02)        | 2.91 (0.08)        | 0.77 (0.01)        | 28.17 (0.81) |
|   | relaxed lasso   | 0.62 (0.02)        | 7.16 (0.63)         | 6.79         | 0.37        | 1.3 (0.03)         | 1.76 (0.07)        | 0.63 (0.02)        | 11.42 (0.69) |
|   | mcp             | 0.85 (0.01)        | 2.15 (0.26)         | 2.07         | 0.08        | 1.12 (0.03)        | 0.77 (0.07)        | 0.34 (0.02)        | 6.99 (0.26)  |
|   | scad            | 0.63 (0.01)        | 6.61 (0.37)         | 6.58         | 0.03        | 1.13 (0.03)        | 0.9 (0.06)         | 0.36 (0.02)        | 11.55 (0.38) |
|   | vsd             | 0.92 (0.01)        | 0.7 (0.08)          | 0.05         | 0.65        | 1.21 (0.03)        | 0.87 (0.06)        | 0.5 (0.03)         | 4.4 (0.08)   |
|   | bic             | 0.81 (0.01)        | 2.79 (0.3)          | 2.73         | 0.06        | 1.13 (0.03)        | 0.82 (0.08)        | 0.35 (0.02)        | 7.67 (0.31)  |
|   | <b>ebic</b>     | 0.85 (0.01)        | 2.15 (0.26)         | 2.07         | 0.08        | <b>1.12</b> (0.03) | <b>0.77</b> (0.07) | <b>0.34</b> (0.02) | 6.99 (0.26)  |
|   | <b>cv</b>       | <u>0.69</u> (0.02) | <u>5.62</u> (0.52)  | <u>5.56</u>  | <b>0.06</b> | 1.13 (0.03)        | 0.9 (0.07)         | 0.36 (0.02)        | 10.5 (0.53)  |
|   | <b>csuv.m.0</b> | <b>0.92</b> (0.01) | <b>0.76</b> (0.08)  | <b>0.23</b>  | <u>0.53</u> | 1.2 (0.03)         | 0.87 (0.06)        | 0.48 (0.03)        | 4.7 (0.09)   |
|   | <b>csuv.s.0</b> | 0.85 (0.01)        | 1.72 (0.13)         | 1.42         | 0.3         | <u>1.21</u> (0.03) | <u>1.06</u> (0.05) | <u>0.51</u> (0.02) | 6.12 (0.15)  |
|   | csuv.m.5        | 0.92 (0.01)        | 0.75 (0.08)         | 0.17         | 0.58        | 1.2 (0.03)         | 0.88 (0.05)        | 0.5 (0.03)         | 4.59 (0.08)  |
|   | csuv.s.5        | 0.81 (0.01)        | 2.35 (0.12)         | 2.08         | 0.27        | 1.22 (0.03)        | 1.17 (0.05)        | 0.53 (0.02)        | 6.81 (0.16)  |
|   | csuv.m.0.all    | 0.91 (0.01)        | 0.83 (0.09)         | 0.31         | 0.52        | 1.2 (0.03)         | 0.89 (0.06)        | 0.49 (0.03)        | 4.79 (0.1)   |
|   | csuv.s.0.all    | 0.83 (0.01)        | 1.98 (0.14)         | 1.7          | 0.28        | 1.21 (0.03)        | 1.1 (0.05)         | 0.51 (0.02)        | 6.42 (0.16)  |
| csuv.m.0.mcp                                      | 0.92 (0.01)     | 0.72 (0.08)        | 0.07                | 0.65         | 1.21 (0.03) | 0.89 (0.05)        | 0.51 (0.03)        | 4.42 (0.07)        |              |
| csuv.s.0.mcp                                      | 0.9 (0.01)      | 0.99 (0.08)        | 0.5                 | 0.49         | 1.2 (0.03)  | 0.92 (0.05)        | 0.5 (0.03)         | 5.01 (0.12)        |              |
| setting 2<br>block.cor = 0.5<br>s = 5<br>p = 300  | lasso           | 0.29 (0.01)        | 26.37 (1.1)         | 26.32        | 0.05        | 1.49 (0.03)        | 3.19 (0.1)         | 0.85 (0.02)        | 31.27 (1.1)  |
|   | elastic net     | 0.23 (0.01)        | 35.16 (1.2)         | 35.1         | 0.06        | 1.62 (0.03)        | 3.94 (0.11)        | 0.98 (0.02)        | 40.04 (1.2)  |
|   | relaxed lasso   | 0.49 (0.01)        | 12.49 (0.9)         | 12.23        | 0.26        | 1.48 (0.03)        | 2.68 (0.12)        | 0.8 (0.03)         | 16.97 (0.94) |
|   | mcp             | 0.79 (0.01)        | 2.99 (0.25)         | 2.9          | 0.09        | 1.15 (0.02)        | 0.93 (0.06)        | 0.41 (0.02)        | 7.81 (0.24)  |
|   | scad            | 0.54 (0.01)        | 9.86 (0.51)         | 9.83         | 0.03        | 1.15 (0.02)        | 1.13 (0.05)        | 0.41 (0.02)        | 14.8 (0.5)   |
|   | vsd             | 0.9 (0.01)         | 1.06 (0.14)         | 0.05         | 1.01        | 1.56 (0.09)        | 1.26 (0.12)        | 0.67 (0.06)        | 4.04 (0.14)  |
|   | bic             | 0.65 (0.02)        | 8.15 (1.17)         | 8.08         | 0.07        | 1.18 (0.03)        | 1.3 (0.12)         | 0.45 (0.02)        | 13.01 (1.18) |
|   | <b>ebic</b>     | 0.79 (0.02)        | 3.75 (0.85)         | 3.66         | 0.09        | <b>1.16</b> (0.02) | 1.01 (0.1)         | <b>0.42</b> (0.02) | 8.57 (0.85)  |
|   | <b>cv</b>       | <u>0.5</u> (0.02)  | <u>14.67</u> (1.34) | <u>14.61</u> | <b>0.06</b> | 1.28 (0.03)        | <u>1.85</u> (0.15) | 0.57 (0.03)        | 19.55 (1.34) |
|   | <b>csuv.m.0</b> | <b>0.92</b> (0.01) | <b>0.72</b> (0.1)   | <b>0.18</b>  | <u>0.54</u> | <u>1.3</u> (0.05)  | <b>0.96</b> (0.08) | 0.52 (0.04)        | 4.64 (0.07)  |
|   | <b>csuv.s.0</b> | 0.73 (0.01)        | 3.8 (0.21)          | 3.67         | 0.13        | 1.28 (0.03)        | 1.55 (0.07)        | <u>0.6</u> (0.02)  | 8.54 (0.2)   |
|   | csuv.m.5        | 0.92 (0.01)        | 0.71 (0.1)          | 0.16         | 0.55        | 1.3 (0.05)         | 0.97 (0.08)        | 0.53 (0.04)        | 4.61 (0.08)  |
|   | csuv.s.5        | 0.67 (0.01)        | 5.08 (0.17)         | 4.96         | 0.12        | 1.32 (0.02)        | 1.8 (0.06)         | 0.65 (0.02)        | 9.84 (0.18)  |
|   | csuv.m.0.all    | 0.92 (0.01)        | 0.72 (0.11)         | 0.18         | 0.54        | 1.29 (0.05)        | 0.96 (0.08)        | 0.52 (0.04)        | 4.64 (0.07)  |
|   | csuv.s.0.all    | 0.72 (0.01)        | 4.13 (0.23)         | 4.01         | 0.12        | 1.28 (0.02)        | 1.6 (0.07)         | 0.6 (0.02)         | 8.89 (0.22)  |
| csuv.m.0.mcp                                      | 0.9 (0.01)      | 0.84 (0.09)        | 0.04                | 0.8          | 1.4 (0.05)  | 1.09 (0.07)        | 0.62 (0.04)        | 4.24 (0.09)        |              |
| csuv.s.0.mcp                                      | 0.87 (0.01)     | 1.38 (0.1)         | 1.12                | 0.26         | 1.23 (0.04) | 1.04 (0.07)        | 0.5 (0.03)         | 5.86 (0.08)        |              |
| setting 3<br>block.cor = 0.5<br>s = 10<br>p = 100 | lasso           | 0.48 (0.01)        | 22.55 (0.82)        | 22.53        | 0.02        | 1.52 (0.03)        | 3.77 (0.1)         | 0.88 (0.01)        | 32.51 (0.83) |
|   | elastic net     | 0.44 (0.01)        | 26.43 (0.83)        | 26.41        | 0.02        | 1.58 (0.03)        | 4.22 (0.1)         | 0.94 (0.01)        | 36.39 (0.83) |
|   | relaxed lasso   | 0.66 (0.01)        | 11.12 (0.69)        | 10.68        | 0.44        | 1.49 (0.02)        | 3.17 (0.1)         | 0.87 (0.02)        | 20.24 (0.75) |
|   | mcp             | 0.89 (0.01)        | 2.62 (0.21)         | 2.44         | 0.18        | 1.22 (0.02)        | 1.5 (0.07)         | 0.54 (0.02)        | 12.26 (0.23) |
|   | scad            | 0.76 (0.01)        | 6.64 (0.3)          | 6.5          | 0.14        | 1.21 (0.02)        | 1.55 (0.06)        | 0.52 (0.02)        | 16.36 (0.3)  |
|   | vsd             | 0.94 (0.01)        | 1.17 (0.11)         | 0.15         | 1.02        | 1.39 (0.03)        | 1.77 (0.08)        | 0.73 (0.03)        | 9.13 (0.1)   |
|   | bic             | 0.87 (0.01)        | 3.28 (0.27)         | 3.12         | 0.16        | 1.22 (0.02)        | 1.52 (0.07)        | 0.54 (0.02)        | 12.96 (0.27) |
|   | <b>ebic</b>     | 0.89 (0.01)        | 2.63 (0.21)         | 2.45         | 0.18        | <b>1.22</b> (0.02) | <b>1.5</b> (0.07)  | <b>0.54</b> (0.02) | 12.27 (0.23) |
|   | <b>cv</b>       | <u>0.5</u> (0.01)  | <u>21.91</u> (0.89) | <u>21.86</u> | <b>0.05</b> | <u>1.52</u> (0.03) | <u>3.72</u> (0.11) | <u>0.88</u> (0.02) | 31.81 (0.9)  |
|   | <b>csuv.m.0</b> | <b>0.93</b> (0.01) | <b>1.39</b> (0.11)  | <b>0.54</b>  | <u>0.85</u> | 1.36 (0.03)        | 1.8 (0.07)         | 0.73 (0.03)        | 9.69 (0.11)  |
|   | <b>csuv.s.0</b> | 0.82 (0.01)        | 4.47 (0.18)         | 4.11         | 0.36        | 1.36 (0.03)        | 2.25 (0.07)        | 0.73 (0.02)        | 13.75 (0.19) |
|   | csuv.m.5        | 0.93 (0.01)        | 1.42 (0.12)         | 0.47         | 0.95        | 1.39 (0.03)        | 1.86 (0.07)        | 0.76 (0.03)        | 9.52 (0.1)   |
|   | csuv.s.5        | 0.8 (0.01)         | 4.86 (0.15)         | 4.54         | 0.32        | 1.37 (0.03)        | 2.29 (0.07)        | 0.73 (0.02)        | 14.22 (0.16) |
|   | csuv.m.0.all    | 0.93 (0.01)        | 1.46 (0.12)         | 0.66         | 0.8         | 1.35 (0.03)        | 1.81 (0.07)        | 0.73 (0.03)        | 9.86 (0.11)  |
|   | csuv.s.0.all    | 0.8 (0.01)         | 5.02 (0.19)         | 4.69         | 0.33        | 1.37 (0.03)        | 2.32 (0.07)        | 0.74 (0.02)        | 14.36 (0.2)  |
| csuv.m.0.mcp                                      | 0.89 (0.01)     | 1.87 (0.12)        | 0.05                | 1.82         | 1.63 (0.04) | 2.34 (0.09)        | 0.97 (0.03)        | 8.23 (0.11)        |              |
| csuv.s.0.mcp                                      | 0.93 (0.01)     | 1.43 (0.1)         | 0.59                | 0.84         | 1.35 (0.03) | 1.8 (0.07)         | 0.73 (0.03)        | 9.75 (0.1)         |              |

Table 5: Model 3: performance of CSUV and methods it compares with. Variable selection performance in terms of F-measure (f), total error (FP+FN), false positives (FP) and false negatives (FN), prediction error in terms of mse (pred.err) and estimation error in terms of l1 and l2 distance (l1.diff and l2.diff) and are shown. The numbers are based on 100 simulations. The last 8 rows are the performance of CSUV with different parameters (e.g. csuv.m.0.mcp corresponds to CSUV with MCP as constituent method and  $r = 0$ ). A bold number represents the best result among delete- $n/2$  cross validation, eBIC and CSUV using Lasso, MCP and SCAD while a underlined number represents the worst among them. Standard errors are shown inside the parentheses.

| parameters  | methods         | f                  | FP+FN               | FP           | FN          | pred.err           | l1.diff            | l2.diff            | size         |
|---|-----------------|--------------------|---------------------|--------------|-------------|--------------------|--------------------|--------------------|--------------|
| setting 4<br>block.cor = 0.5<br>s = 10<br>p = 300 | lasso           | 0.37 (0.01)        | 35.15 (1.15)        | 35.03        | 0.12        | 1.77 (0.04)        | 5.29 (0.13)        | 1.11 (0.02)        | 44.91 (1.15) |
|   | elastic net     | 0.31 (0.01)        | 45.8 (1.39)         | 45.65        | 0.15        | 1.97 (0.05)        | 6.59 (0.17)        | 1.27 (0.02)        | 55.5 (1.39)  |
|   | relaxed lasso   | 0.51 (0.01)        | 20.03 (0.83)        | 19.75        | 0.28        | 1.73 (0.04)        | 4.62 (0.15)        | 1.04 (0.03)        | 29.47 (0.86) |
|   | mcp             | 0.84 (0.01)        | 3.71 (0.27)         | 3.31         | 0.4         | 1.39 (0.06)        | 1.96 (0.14)        | 0.67 (0.04)        | 12.91 (0.21) |
|   | scad            | 0.68 (0.01)        | 9.88 (0.44)         | 9.75         | 0.13        | 1.3 (0.04)         | 1.96 (0.11)        | 0.62 (0.03)        | 19.62 (0.41) |
|   | vsd             | 0.91 (0.01)        | 1.83 (0.2)          | 0.21         | 1.62        | 1.88 (0.17)        | 2.31 (0.16)        | 0.92 (0.06)        | 8.59 (0.18)  |
|   | bic             | 0.73 (0.02)        | 10.55 (1.62)        | 10.37        | 0.18        | 1.37 (0.04)        | 2.35 (0.19)        | 0.68 (0.04)        | 20.19 (1.62) |
|   | <b>ebic</b>     | <b>0.85</b> (0.01) | 4.28 (0.81)         | 3.97         | 0.31        | <b>1.33</b> (0.04) | <b>1.9</b> (0.13)  | <b>0.64</b> (0.03) | 13.66 (0.81) |
|   | <b>cv</b>       | <u>0.37</u> (0.01) | <u>35.15</u> (1.15) | <u>35.03</u> | <b>0.12</b> | 1.77 (0.04)        | <u>5.29</u> (0.13) | <u>1.11</u> (0.02) | 44.91 (1.15) |
|   | <b>csuv.m.0</b> | <b>0.85</b> (0.01) | <b>2.77</b> (0.17)  | <b>0.88</b>  | <u>1.89</u> | <u>1.92</u> (0.07) | 2.73 (0.12)        | 1.08 (0.04)        | 8.99 (0.13)  |
|   | <b>csuv.s.0</b> | 0.61 (0.01)        | 12.58 (0.35)        | 12.06        | 0.52        | 1.64 (0.04)        | 3.82 (0.11)        | 1 (0.02)           | 21.54 (0.37) |
|   | csuv.m.5        | 0.85 (0.01)        | 2.78 (0.17)         | 0.73         | 2.05        | 2.01 (0.08)        | 2.79 (0.13)        | 1.11 (0.04)        | 8.68 (0.13)  |
|   | csuv.s.5        | 0.6 (0.01)         | 13 (0.35)           | 12.45        | 0.55        | 1.66 (0.04)        | 3.89 (0.12)        | 1 (0.03)           | 21.9 (0.37)  |
|   | csuv.m.0.all    | 0.85 (0.01)        | 2.85 (0.18)         | 1.12         | 1.73        | 1.87 (0.07)        | 2.65 (0.12)        | 1.04 (0.04)        | 9.39 (0.14)  |
|   | csuv.s.0.all    | 0.58 (0.01)        | 14.46 (0.41)        | 13.95        | 0.51        | 1.68 (0.04)        | 4.11 (0.12)        | 1.02 (0.03)        | 23.44 (0.44) |
| csuv.m.0.mcp                                      | 0.6 (0.02)      | 5.53 (0.16)        | 0.04                | 5.49         | 4.62 (0.2)  | 5.23 (0.13)        | 1.9 (0.04)         | 4.55 (0.16)        |              |
| csuv.s.0.mcp                                      | 0.82 (0.01)     | 3.47 (0.21)        | 1.37                | 2.1          | 2.07 (0.08) | 3.14 (0.15)        | 1.18 (0.04)        | 9.27 (0.07)        |              |
| setting 5<br>block.cor = 0.9<br>s = 5<br>p = 100  | lasso           | 0.38 (0.01)        | 14.34 (0.77)        | 13.22        | 1.12        | 1.27 (0.02)        | 4.03 (0.14)        | 1.28 (0.02)        | 17.1 (0.85)  |
|   | elastic net     | 0.31 (0.01)        | 19.43 (0.84)        | 18.45        | 0.98        | 1.3 (0.02)         | 4.77 (0.15)        | 1.38 (0.02)        | 22.47 (0.91) |
|   | relaxed lasso   | 0.48 (0.01)        | 8.15 (0.5)          | 6.46         | 1.69        | 1.26 (0.02)        | 3.42 (0.11)        | 1.26 (0.03)        | 9.77 (0.6)   |
|   | mcp             | 0.58 (0.02)        | 4.88 (0.22)         | 3.03         | 1.85        | 1.28 (0.02)        | 2.99 (0.15)        | 1.34 (0.05)        | 6.18 (0.26)  |
|   | scad            | 0.6 (0.02)         | 4.7 (0.27)          | 3.04         | 1.66        | 1.25 (0.02)        | 2.7 (0.15)         | 1.26 (0.05)        | 6.38 (0.32)  |
|   | vsd             | 0.65 (0.02)        | 2.86 (0.14)         | 0.46         | 2.4         | 1.25 (0.03)        | 2.41 (0.13)        | 1.25 (0.04)        | 3.06 (0.03)  |
|   | bic             | 0.61 (0.02)        | 4.35 (0.21)         | 2.7          | 1.65        | 1.26 (0.03)        | 2.74 (0.15)        | 1.26 (0.05)        | 6.05 (0.29)  |
|   | <b>ebic</b>     | 0.62 (0.02)        | 4.13 (0.2)          | 2.39         | 1.74        | <u>1.26</u> (0.03) | 2.66 (0.14)        | 1.24 (0.05)        | 5.65 (0.28)  |
|   | <b>cv</b>       | <u>0.47</u> (0.02) | <u>10.44</u> (0.76) | <u>9.07</u>  | <b>1.37</b> | <u>1.26</u> (0.02) | <u>3.53</u> (0.16) | <u>1.28</u> (0.03) | 12.7 (0.86)  |
|   | <b>csuv.m.0</b> | <b>0.66</b> (0.01) | <b>2.99</b> (0.12)  | <b>0.85</b>  | <u>2.14</u> | 1.25 (0.03)        | <b>2.41</b> (0.09) | <b>1.19</b> (0.03) | 3.71 (0.09)  |
|   | <b>csuv.s.0</b> | 0.58 (0.01)        | 4.72 (0.16)         | 2.89         | 1.83        | <b>1.22</b> (0.02) | 2.95 (0.09)        | 1.22 (0.03)        | 6.06 (0.14)  |
|   | csuv.m.5        | 0.66 (0.01)        | 2.93 (0.12)         | 0.74         | 2.19        | 1.25 (0.03)        | 2.37 (0.09)        | 1.19 (0.03)        | 3.55 (0.09)  |
|   | csuv.s.5        | 0.57 (0.01)        | 4.88 (0.14)         | 3.03         | 1.85        | 1.23 (0.02)        | 2.98 (0.09)        | 1.23 (0.03)        | 6.18 (0.13)  |
|   | csuv.m.0.all    | 0.63 (0.01)        | 3.42 (0.16)         | 1.28         | 2.14        | 1.25 (0.03)        | 2.6 (0.1)          | 1.22 (0.03)        | 4.14 (0.11)  |
|   | csuv.s.0.all    | 0.53 (0.01)        | 5.71 (0.17)         | 3.91         | 1.8         | 1.23 (0.02)        | 3.16 (0.09)        | 1.25 (0.03)        | 7.11 (0.16)  |
| csuv.m.0.mcp                                      | 0.53 (0.02)     | 3.51 (0.12)        | 0.18                | 3.33         | 2.42 (0.1)  | 3.1 (0.11)         | 1.56 (0.04)        | 1.85 (0.09)        |              |
| csuv.s.0.mcp                                      | 0.61 (0.02)     | 3.45 (0.18)        | 0.99                | 2.46         | 1.39 (0.04) | 2.91 (0.15)        | 1.39 (0.05)        | 3.53 (0.08)        |              |
| setting 6<br>block.cor = 0.9<br>s = 5<br>p = 300  | lasso           | 0.28 (0.01)        | 21.2 (1.05)         | 19.8         | 1.4         | 1.4 (0.02)         | 5.37 (0.15)        | 1.6 (0.02)         | 23.4 (1.1)   |
|   | elastic net     | 0.21 (0.01)        | 30.54 (1.35)        | 29.32        | 1.22        | 1.41 (0.02)        | 6.18 (0.2)         | 1.66 (0.02)        | 33.1 (1.41)  |
|   | relaxed lasso   | 0.34 (0.01)        | 14.1 (0.77)         | 12.28        | 1.82        | 1.41 (0.03)        | 5.18 (0.17)        | 1.65 (0.03)        | 15.46 (0.83) |
|   | mcp             | 0.42 (0.02)        | 7.21 (0.26)         | 4.52         | 2.69        | 1.45 (0.03)        | 4.8 (0.16)         | 1.85 (0.04)        | 6.83 (0.25)  |
|   | scad            | 0.4 (0.02)         | 8.83 (0.35)         | 6.53         | 2.3         | 1.42 (0.03)        | 4.42 (0.19)        | 1.74 (0.05)        | 9.23 (0.32)  |
|   | vsd             | 0.45 (0.02)        | 4.8 (0.14)          | 1.14         | 3.66        | 2 (0.06)           | 4.4 (0.11)         | 1.94 (0.03)        | 2.48 (0.12)  |
|   | bic             | 0.42 (0.02)        | 9.21 (0.91)         | 6.9          | 2.31        | 1.44 (0.03)        | 4.76 (0.21)        | 1.77 (0.05)        | 9.59 (0.95)  |
|   | <b>ebic</b>     | 0.44 (0.02)        | 6.91 (0.27)         | 4.39         | 2.52        | 1.43 (0.03)        | 4.59 (0.17)        | <u>1.79</u> (0.05) | 6.87 (0.26)  |
|   | <b>cv</b>       | <u>0.29</u> (0.01) | <u>20.18</u> (1.01) | <u>18.7</u>  | <b>1.48</b> | 1.39 (0.02)        | <u>5.31</u> (0.15) | <b>1.62</b> (0.02) | 22.22 (1.06) |
|   | <b>csuv.m.0</b> | <b>0.51</b> (0.02) | <b>3.97</b> (0.12)  | <b>0.81</b>  | <u>3.16</u> | <u>2.06</u> (0.06) | <b>3.78</b> (0.1)  | 1.72 (0.03)        | 2.65 (0.12)  |
|   | <b>csuv.s.0</b> | 0.44 (0.01)        | 7.26 (0.19)         | 5.08         | 2.18        | <b>1.38</b> (0.03) | 4.45 (0.11)        | 1.65 (0.03)        | 7.9 (0.13)   |
|   | csuv.m.5        | 0.51 (0.02)        | 3.86 (0.12)         | 0.65         | 3.21        | 2.1 (0.06)         | 3.71 (0.1)         | 1.71 (0.03)        | 2.44 (0.11)  |
|   | csuv.s.5        | 0.44 (0.01)        | 7.46 (0.19)         | 5.32         | 2.14        | 1.38 (0.03)        | 4.45 (0.1)         | 1.64 (0.03)        | 8.18 (0.13)  |
|   | csuv.m.0.all    | 0.51 (0.02)        | 4.02 (0.13)         | 1.02         | 3           | 1.94 (0.06)        | 3.8 (0.1)          | 1.7 (0.03)         | 3.02 (0.13)  |
|   | csuv.s.0.all    | 0.42 (0.01)        | 8.34 (0.2)          | 6.34         | 2           | 1.37 (0.03)        | 4.51 (0.1)         | 1.62 (0.03)        | 9.34 (0.17)  |
| csuv.m.0.mcp                                      | 0.42 (0.01)     | 4.26 (0.09)        | 0.14                | 4.12         | 2.89 (0.06) | 3.93 (0.07)        | 1.84 (0.02)        | 1.02 (0.09)        |              |
| csuv.s.0.mcp                                      | 0.47 (0.02)     | 5.21 (0.17)        | 2.35                | 2.86         | 1.65 (0.05) | 4.39 (0.11)        | 1.78 (0.03)        | 4.49 (0.08)        |              |

Table 6: Model 3: performance of CSUV and methods it compares with (continue). Variable selection performance in terms of F-measure (f), total error (FP+FN), false positives (FP) and false negatives (FN), prediction error in terms of mse (pred.err) and estimation error in terms of l1 and l2 distance (l1.diff and l2.diff) and are shown. The numbers are based on 100 simulations. The last 8 rows are the performance of CSUV with different parameters (e.g. csuv.m.0.mcp corresponds to CSUV with MCP as constituent method and  $r = 0$ ). A bold number represents the best result among delete- $n/2$  cross validation, eBIC and CSUV using Lasso, MCP and SCAD while a underlined number represents the worst among them. Standard errors are shown inside the parentheses.

| parameters  | methods         | f                  | FP+FN               | FP           | FN          | pred.err           | l1.diff             | l2.diff            | size         |
|---|-----------------|--------------------|---------------------|--------------|-------------|--------------------|---------------------|--------------------|--------------|
| setting 7<br>block.cor = 0.9<br>s = 10<br>p = 100 | lasso           | 0.5 (0.01)         | 18.54 (0.66)        | 17.19        | 1.35        | 1.42 (0.02)        | 6.55 (0.16)         | 1.65 (0.03)        | 25.84 (0.69) |
|   | elastic net     | 0.44 (0.01)        | 22.85 (0.63)        | 21.67        | 1.18        | 1.43 (0.02)        | 7.18 (0.15)         | 1.7 (0.02)         | 30.49 (0.67) |
|   | relaxed lasso   | 0.56 (0.01)        | 13.74 (0.64)        | 11.91        | 1.83        | 1.42 (0.03)        | 6.37 (0.18)         | 1.68 (0.03)        | 20.08 (0.72) |
|   | mcp             | 0.53 (0.01)        | 8.51 (0.29)         | 3.35         | 5.16        | 1.73 (0.04)        | 7.19 (0.25)         | 2.34 (0.06)        | 8.19 (0.13)  |
|   | scad            | 0.59 (0.01)        | 7.76 (0.28)         | 3.23         | 4.53        | 1.61 (0.04)        | 6.23 (0.21)         | 2.17 (0.06)        | 8.7 (0.16)   |
|   | vsd             | 0.6 (0.01)         | 6.52 (0.23)         | 1.53         | 4.99        | 1.8 (0.06)         | 6.56 (0.22)         | 2.3 (0.06)         | 6.54 (0.1)   |
|   | bic             | 0.6 (0.01)         | 7.5 (0.28)          | 3.04         | 4.46        | 1.59 (0.04)        | 6.11 (0.21)         | 2.11 (0.06)        | 8.58 (0.21)  |
|   | <b>ebic</b>     | 0.6 (0.01)         | 7.27 (0.27)         | 2.77         | <u>4.5</u>  | 1.6 (0.04)         | 6.12 (0.22)         | <u>2.12</u> (0.06) | 8.27 (0.14)  |
|   | <b>cv</b>       | <u>0.5</u> (0.01)  | <u>18.54</u> (0.66) | <u>17.19</u> | <b>1.35</b> | 1.42 (0.02)        | 6.55 (0.16)         | <b>1.65</b> (0.03) | 25.84 (0.69) |
|   | <b>csuv.m.0</b> | <b>0.69</b> (0.01) | <b>5.88</b> (0.23)  | <b>2.47</b>  | 3.41        | <u>1.66</u> (0.05) | <b>5.24</b> (0.16)  | 1.77 (0.04)        | 9.06 (0.18)  |
|   | <b>csuv.s.0</b> | 0.62 (0.01)        | 9.49 (0.27)         | 7.22         | 2.27        | <b>1.38</b> (0.02) | 5.69 (0.14)         | <b>1.65</b> (0.03) | 14.95 (0.22) |
|   | csuv.m.5        | 0.68 (0.01)        | 5.92 (0.23)         | 2.38         | 3.54        | 1.71 (0.06)        | 5.34 (0.16)         | 1.8 (0.04)         | 8.84 (0.18)  |
|   | csuv.s.5        | 0.62 (0.01)        | 9.76 (0.26)         | 7.54         | 2.22        | 1.38 (0.02)        | 5.67 (0.13)         | 1.64 (0.03)        | 15.32 (0.21) |
|   | csuv.m.0.all    | 0.7 (0.01)         | 5.99 (0.25)         | 2.86         | 3.13        | 1.58 (0.04)        | 5.21 (0.17)         | 1.73 (0.04)        | 9.73 (0.18)  |
|   | csuv.s.0.all    | 0.6 (0.01)         | 10.58 (0.28)        | 8.54         | 2.04        | 1.38 (0.02)        | 5.79 (0.13)         | 1.64 (0.03)        | 16.5 (0.25)  |
|   | csuv.m.0.mcp    | 0.4 (0.01)         | 7.64 (0.13)         | 0.31         | 7.33        | 5.38 (0.28)        | 7.79 (0.14)         | 2.71 (0.04)        | 2.98 (0.12)  |
|   | csuv.s.0.mcp    | 0.6 (0.01)         | 7.01 (0.21)         | 2.25         | 4.76        | 1.98 (0.06)        | 6.57 (0.19)         | 2.22 (0.05)        | 7.49 (0.08)  |
| setting 8<br>block.cor = 0.9<br>s = 10<br>p = 300 | lasso           | 0.33 (0.01)        | 34.07 (0.92)        | 32.27        | 1.8         | 1.59 (0.03)        | 10.03 (0.23)        | 2.1 (0.03)         | 40.47 (0.9)  |
|   | elastic net     | 0.29 (0.01)        | 42.66 (0.87)        | 41.11        | 1.55        | 1.62 (0.03)        | 10.86 (0.21)        | 2.15 (0.03)        | 49.56 (0.84) |
|   | relaxed lasso   | 0.35 (0.01)        | 28.37 (0.78)        | 25.88        | 2.49        | 1.63 (0.03)        | 10.14 (0.22)        | 2.19 (0.04)        | 33.39 (0.88) |
|   | mcp             | 0.38 (0.02)        | 12.94 (0.35)        | 6.81         | 6.13        | 2.2 (0.06)         | 11.21 (0.31)        | 3.09 (0.06)        | 10.68 (0.18) |
|   | scad            | 0.43 (0.02)        | 12.2 (0.39)         | 6.8          | 5.4         | 2.01 (0.05)        | 9.94 (0.32)         | 2.89 (0.07)        | 11.4 (0.21)  |
|   | vsd             | 0.44 (0.02)        | 10 (0.31)           | 3.78         | 6.22        | 2.73 (0.17)        | 9.8 (0.29)          | 2.97 (0.06)        | 7.56 (0.14)  |
|   | bic             | 0.42 (0.02)        | 14 (1)              | 8.7          | 5.3         | 2.05 (0.05)        | 10.53 (0.34)        | 2.92 (0.07)        | 13.4 (1.09)  |
|   | <b>ebic</b>     | 0.44 (0.02)        | 11.7 (0.35)         | 6.29         | 5.41        | 2.03 (0.05)        | <u>10.05</u> (0.31) | <u>2.9</u> (0.06)  | 10.88 (0.18) |
|   | <b>cv</b>       | <u>0.33</u> (0.01) | <u>34.07</u> (0.92) | <u>32.27</u> | <b>1.8</b>  | <b>1.59</b> (0.03) | 10.03 (0.23)        | <b>2.1</b> (0.03)  | 40.47 (0.9)  |
|   | <b>csuv.m.0</b> | <b>0.52</b> (0.01) | <b>7.84</b> (0.22)  | <b>2.15</b>  | <u>5.69</u> | <u>3.96</u> (0.16) | <b>7.74</b> (0.21)  | 2.52 (0.05)        | 6.46 (0.16)  |
|   | <b>csuv.s.0</b> | 0.42 (0.01)        | 20.14 (0.32)        | 17.25        | 2.89        | 1.65 (0.03)        | 9.4 (0.2)           | 2.21 (0.04)        | 24.36 (0.25) |
|   | csuv.m.5        | 0.52 (0.01)        | 7.69 (0.21)         | 1.87         | 5.82        | 4.29 (0.16)        | 7.61 (0.2)          | 2.5 (0.05)         | 6.05 (0.15)  |
|   | csuv.s.5        | 0.41 (0.01)        | 20.32 (0.32)        | 17.36        | 2.96        | 1.67 (0.03)        | 9.45 (0.2)          | 2.21 (0.04)        | 24.4 (0.23)  |
|   | csuv.m.0.all    | 0.53 (0.01)        | 7.91 (0.21)         | 2.46         | 5.45        | 3.55 (0.15)        | 7.77 (0.19)         | 2.53 (0.05)        | 7.01 (0.16)  |
|   | csuv.s.0.all    | 0.41 (0.01)        | 21.06 (0.34)        | 18.27        | 2.79        | 1.65 (0.03)        | 9.47 (0.2)          | 2.19 (0.04)        | 25.48 (0.26) |
|   | csuv.m.0.mcp    | 0.23 (0.01)        | 9.67 (0.09)         | 0.22         | 9.45        | 10.76 (0.22)       | 8.96 (0.1)          | 2.97 (0.03)        | 0.77 (0.09)  |
|   | csuv.s.0.mcp    | 0.36 (0.02)        | 11.97 (0.28)        | 5.33         | 6.64        | 3.41 (0.12)        | 10.68 (0.25)        | 3.08 (0.05)        | 8.69 (0.05)  |

Table 7: Model 3: performance of CSUV and methods it compares with (continue). Variable selection performance in terms of F-measure (f), total error (FP+FN), false positives (FP) and false negatives (FN), prediction error in terms of mse (pred.err) and estimation error in terms of l1 and l2 distance (l1.diff and l2.diff) and are shown. The numbers are based on 100 simulations. The last 8 rows are the performance of CSUV with different parameters (e.g. csuv.m.0.mcp corresponds to CSUV with MCP as constituent method and  $r = 0$ ). A bold number represents the best result among delete- $n/2$  cross validation, eBIC and CSUV using Lasso, MCP and SCAD while a underlined number represents the worst among them. Standard errors are shown inside the parentheses.

| parameters                                       | methods         | f                  | FP+FN               | FP           | FN          | pred.err           | l1.diff            | l2.diff            | size         |
|--|-----------------|--------------------|---------------------|--------------|-------------|--------------------|--------------------|--------------------|--------------|
| setting 1<br>num.factor = 2<br>s = 5<br>p = 100  | lasso           | 0.37 (0.01)        | 18.19 (0.89)        | 17.98        | 0.21        | 1.85 (0.09)        | 3.02 (0.13)        | 0.9 (0.03)         | 22.77 (0.89) |
|  | elastic net     | 0.32 (0.01)        | 23.2 (0.97)         | 23.01        | 0.19        | 1.98 (0.08)        | 3.57 (0.13)        | 0.98 (0.03)        | 27.82 (0.98) |
|  | relaxed lasso   | 0.5 (0.02)         | 11.16 (0.77)        | 10.68        | 0.48        | 1.89 (0.08)        | 2.77 (0.12)        | 0.91 (0.03)        | 15.2 (0.81)  |
|  | mcp             | 0.75 (0.02)        | 3.26 (0.28)         | 2.57         | 0.69        | 1.71 (0.09)        | 1.67 (0.11)        | 0.76 (0.04)        | 6.88 (0.26)  |
|  | scad            | 0.65 (0.02)        | 5.65 (0.37)         | 5.17         | 0.48        | 1.67 (0.09)        | 1.62 (0.1)         | 0.73 (0.04)        | 9.69 (0.37)  |
|  | vsd             | 0.82 (0.02)        | 1.71 (0.15)         | 0.19         | 1.52        | 2.22 (0.15)        | 1.9 (0.13)         | 0.98 (0.06)        | 3.67 (0.13)  |
|  | bic             | 0.72 (0.02)        | 4.01 (0.32)         | 3.41         | 0.6         | 1.69 (0.1)         | 1.67 (0.11)        | 0.74 (0.04)        | 7.81 (0.31)  |
|  | <b>ebic</b>     | 0.75 (0.02)        | 3.18 (0.28)         | 2.52         | 0.66        | <b>1.7</b> (0.1)   | <b>1.62</b> (0.11) | <b>0.74</b> (0.04) | 6.86 (0.27)  |
|  | <b>cv</b>       | <u>0.47</u> (0.02) | <u>14.49</u> (1.1)  | <u>14.18</u> | <b>0.31</b> | 1.8 (0.09)         | <u>2.66</u> (0.15) | <u>0.85</u> (0.03) | 18.87 (1.12) |
|  | <b>csuv.m.0</b> | <b>0.81</b> (0.02) | <b>1.81</b> (0.15)  | <b>0.82</b>  | <u>0.99</u> | <u>1.95</u> (0.13) | 1.73 (0.11)        | <u>0.85</u> (0.05) | 4.83 (0.13)  |
|  | <b>csuv.s.0</b> | 0.69 (0.01)        | 4.03 (0.23)         | 3.34         | 0.69        | 1.84 (0.09)        | 2.05 (0.1)         | <u>0.85</u> (0.04) | 7.65 (0.23)  |
|  | csuv.m.5        | 0.82 (0.02)        | 1.7 (0.14)          | 0.67         | 1.03        | 1.95 (0.13)        | 1.71 (0.11)        | 0.85 (0.05)        | 4.64 (0.12)  |
|  | csuv.s.5        | 0.67 (0.01)        | 4.43 (0.21)         | 3.73         | 0.7         | 1.84 (0.09)        | 2.12 (0.1)         | 0.86 (0.04)        | 8.03 (0.21)  |
|  | csuv.m.0.all    | 0.81 (0.02)        | 1.93 (0.18)         | 0.98         | 0.95        | 1.87 (0.11)        | 1.71 (0.11)        | 0.83 (0.05)        | 5.03 (0.15)  |
|  | csuv.s.0.all    | 0.66 (0.01)        | 4.67 (0.24)         | 4            | 0.67        | 1.84 (0.09)        | 2.14 (0.1)         | 0.86 (0.04)        | 8.33 (0.26)  |
|  | csuv.m.0.mcp    | 0.78 (0.02)        | 1.73 (0.13)         | 0.15         | 1.58        | 2.32 (0.14)        | 2.03 (0.11)        | 1.04 (0.05)        | 3.57 (0.21)  |
| csuv.s.0.mcp                                     | 0.78 (0.02)     | 2.19 (0.17)        | 1.08                | 1.11         | 2.03 (0.11) | 1.91 (0.11)        | 0.92 (0.05)        | 4.97 (0.1)         |              |
| setting 2<br>num.factor = 2<br>s = 5<br>p = 300  | lasso           | 0.3 (0.01)         | 26.02 (1.59)        | 25.54        | 0.48        | 2.19 (0.08)        | 3.84 (0.15)        | 1.09 (0.03)        | 30.06 (1.64) |
|  | elastic net     | 0.24 (0.01)        | 32.73 (1.45)        | 32.32        | 0.41        | 2.41 (0.09)        | 4.46 (0.14)        | 1.19 (0.03)        | 36.91 (1.49) |
|  | relaxed lasso   | 0.43 (0.01)        | 14.49 (1.11)        | 13.72        | 0.77        | 2.17 (0.09)        | 3.44 (0.13)        | 1.07 (0.03)        | 17.95 (1.18) |
|  | mcp             | 0.68 (0.02)        | 4.14 (0.28)         | 3.19         | 0.95        | 2.02 (0.12)        | 1.94 (0.13)        | 0.89 (0.05)        | 7.24 (0.26)  |
|  | scad            | 0.53 (0.01)        | 8.71 (0.45)         | 8.12         | 0.59        | 1.83 (0.1)         | 1.87 (0.11)        | 0.8 (0.04)         | 12.53 (0.45) |
|  | vsd             | 0.68 (0.02)        | 2.52 (0.15)         | 0.19         | 2.33        | 3.16 (0.22)        | 2.61 (0.14)        | 1.32 (0.06)        | 2.86 (0.13)  |
|  | bic             | 0.57 (0.02)        | 10.8 (1.67)         | 10.12        | 0.68        | 1.91 (0.11)        | 2.35 (0.2)         | 0.86 (0.05)        | 14.44 (1.7)  |
|  | <b>ebic</b>     | 0.66 (0.02)        | 5.83 (1.15)         | 4.94         | 0.89        | <b>2.03</b> (0.13) | <b>2.08</b> (0.16) | <b>0.88</b> (0.05) | 9.05 (1.17)  |
|  | <b>cv</b>       | <u>0.4</u> (0.02)  | <u>20.34</u> (1.67) | <u>19.83</u> | <b>0.51</b> | 2.12 (0.09)        | <u>3.26</u> (0.17) | 1.01 (0.04)        | 24.32 (1.7)  |
|  | <b>csuv.m.0</b> | <b>0.75</b> (0.02) | <b>2.09</b> (0.13)  | <b>0.52</b>  | <u>1.57</u> | <u>2.28</u> (0.12) | 2.11 (0.1)         | <u>1.07</u> (0.05) | 3.95 (0.14)  |
|  | <b>csuv.s.0</b> | 0.62 (0.01)        | 5.37 (0.27)         | 4.44         | 0.93        | 2.05 (0.09)        | 2.49 (0.1)         | 0.99 (0.04)        | 8.51 (0.32)  |
|  | csuv.m.5        | 0.74 (0.02)        | 2.13 (0.14)         | 0.44         | 1.69        | 2.42 (0.13)        | 2.18 (0.11)        | 1.11 (0.05)        | 3.75 (0.13)  |
|  | csuv.s.5        | 0.58 (0.01)        | 6.04 (0.25)         | 5.06         | 0.98        | 2.11 (0.09)        | 2.63 (0.09)        | 1.02 (0.04)        | 9.08 (0.31)  |
|  | csuv.m.0.all    | 0.75 (0.02)        | 2.15 (0.13)         | 0.64         | 1.51        | 2.22 (0.11)        | 2.1 (0.1)          | 1.05 (0.05)        | 4.13 (0.15)  |
|  | csuv.s.0.all    | 0.59 (0.01)        | 6.2 (0.32)          | 5.31         | 0.89        | 2.07 (0.09)        | 2.58 (0.1)         | 1 (0.04)           | 9.42 (0.37)  |
|  | csuv.m.0.mcp    | 0.66 (0.02)        | 2.47 (0.12)         | 0.04         | 2.43        | 2.92 (0.15)        | 2.61 (0.1)         | 1.33 (0.04)        | 2.61 (0.12)  |
| csuv.s.0.mcp                                     | 0.71 (0.02)     | 2.63 (0.15)        | 1.03                | 1.6          | 2.57 (0.15) | 2.41 (0.12)        | 1.17 (0.05)        | 4.43 (0.13)        |              |
| setting 3<br>num.factor = 2<br>s = 10<br>p = 100 | lasso           | 0.45 (0.01)        | 25.15 (0.93)        | 24.76        | 0.39        | 2.6 (0.11)         | 5.16 (0.15)        | 1.23 (0.03)        | 34.37 (0.95) |
|  | elastic net     | 0.41 (0.01)        | 29.57 (0.97)        | 29.24        | 0.33        | 2.73 (0.11)        | 5.84 (0.15)        | 1.3 (0.02)         | 38.91 (0.99) |
|  | relaxed lasso   | 0.56 (0.01)        | 16.19 (0.9)         | 15.26        | 0.93        | 2.74 (0.12)        | 4.82 (0.14)        | 1.25 (0.03)        | 24.33 (1)    |
|  | mcp             | 0.79 (0.01)        | 4.32 (0.24)         | 2.68         | 1.64        | 2.62 (0.16)        | 3.08 (0.16)        | 1.14 (0.05)        | 11.04 (0.22) |
|  | scad            | 0.74 (0.01)        | 6.33 (0.27)         | 5.11         | 1.22        | 2.49 (0.14)        | 3.02 (0.13)        | 1.11 (0.05)        | 13.89 (0.29) |
|  | vsd             | 0.71 (0.02)        | 4.26 (0.24)         | 0.19         | 4.07        | 4.24 (0.29)        | 4.36 (0.19)        | 1.67 (0.06)        | 6.12 (0.24)  |
|  | bic             | 0.79 (0.01)        | 4.77 (0.26)         | 3.47         | 1.3         | 2.47 (0.14)        | 2.96 (0.15)        | 1.09 (0.05)        | 12.17 (0.25) |
|  | <b>ebic</b>     | <b>0.8</b> (0.01)  | 4.32 (0.23)         | 2.85         | 1.47        | <b>2.56</b> (0.14) | <b>2.98</b> (0.15) | <b>1.11</b> (0.05) | 11.38 (0.25) |
|  | <b>cv</b>       | <u>0.47</u> (0.01) | <u>24</u> (1.03)    | <u>23.48</u> | <b>0.52</b> | 2.67 (0.11)        | <u>5.1</u> (0.15)  | 1.24 (0.03)        | 32.96 (1.09) |
|  | <b>csuv.m.0</b> | 0.79 (0.01)        | <b>3.71</b> (0.16)  | <b>1.05</b>  | <u>2.66</u> | <u>3.29</u> (0.17) | 3.66 (0.14)        | <u>1.4</u> (0.04)  | 8.39 (0.2)   |
|  | <b>csuv.s.0</b> | 0.71 (0.01)        | 7.01 (0.29)         | 5.28         | 1.73        | 2.82 (0.14)        | 3.85 (0.12)        | 1.27 (0.04)        | 13.55 (0.34) |
|  | csuv.m.5        | 0.79 (0.01)        | 3.73 (0.17)         | 0.94         | 2.79        | 3.39 (0.18)        | 3.71 (0.14)        | 1.42 (0.05)        | 8.15 (0.19)  |
|  | csuv.s.5        | 0.7 (0.01)         | 7.14 (0.27)         | 5.45         | 1.69        | 2.87 (0.14)        | 3.86 (0.12)        | 1.26 (0.04)        | 13.76 (0.32) |
|  | csuv.m.0.all    | 0.8 (0.01)         | 3.74 (0.17)         | 1.25         | 2.49        | 3.19 (0.18)        | 3.58 (0.13)        | 1.37 (0.04)        | 8.76 (0.21)  |
|  | csuv.s.0.all    | 0.69 (0.01)        | 7.88 (0.33)         | 6.3          | 1.58        | 2.84 (0.14)        | 3.96 (0.13)        | 1.26 (0.04)        | 14.72 (0.37) |
|  | csuv.m.0.mcp    | 0.67 (0.01)        | 4.84 (0.15)         | 0.11         | 4.73        | 5.18 (0.32)        | 5.18 (0.16)        | 1.95 (0.05)        | 5.38 (0.16)  |
| csuv.s.0.mcp                                     | 0.75 (0.01)     | 4.26 (0.19)        | 0.93                | 3.33         | 4.23 (0.29) | 4.28 (0.17)        | 1.61 (0.05)        | 7.6 (0.17)         |              |

Table 8: Model 4: performance of CSUV and methods it compares with. Variable selection performance in terms of F-measure (f), total error (FP+FN), false positives (FP) and false negatives (FN), prediction error in terms of mse (pred.err) and estimation error in terms of l1 and l2 distance (l1.diff and l2.diff) and are shown. The numbers are based on 100 simulations. The last 8 rows are the performance of CSUV with different parameters (e.g. csuv.m.0.mcp corresponds to CSUV with MCP as constituent method and  $r = 0$ ). A bold number represents the best result among delete- $n/2$  cross validation, eBIC and CSUV using Lasso, MCP and SCAD while a underlined number represents the worst among them. Standard errors are shown inside the parentheses.

| parameters                                       | methods         | f                  | FP+FN               | FP           | FN          | pred.err           | l1.diff            | l2.diff            | size         |
|--|-----------------|--------------------|---------------------|--------------|-------------|--------------------|--------------------|--------------------|--------------|
| setting 4<br>num.factor = 2<br>s = 10<br>p = 300 | lasso           | 0.32 (0.01)        | 40.26 (1.61)        | 39.05        | 1.21        | 3.71 (0.18)        | 7.52 (0.2)         | 1.64 (0.03)        | 47.84 (1.67) |
|  | elastic net     | 0.28 (0.01)        | 47.74 (1.62)        | 46.54        | 1.2         | 4.11 (0.19)        | 8.45 (0.19)        | 1.74 (0.03)        | 55.34 (1.68) |
|  | relaxed lasso   | 0.43 (0.01)        | 23.75 (1.41)        | 21.6         | 2.15        | 3.87 (0.22)        | 6.88 (0.21)        | 1.67 (0.04)        | 29.45 (1.59) |
|  | mcp             | 0.67 (0.01)        | 7.02 (0.35)         | 4.06         | 2.96        | 4 (0.26)           | 4.64 (0.2)         | 1.63 (0.06)        | 11.1 (0.36)  |
|  | scad            | 0.58 (0.01)        | 11.98 (0.47)        | 9.91         | 2.07        | 3.48 (0.22)        | 4.36 (0.18)        | 1.49 (0.06)        | 17.84 (0.49) |
|  | vsd             | 0.5 (0.02)         | 7.11 (0.24)         | 0.26         | 6.85        | 6.99 (0.38)        | 6.89 (0.23)        | 2.41 (0.07)        | 3.41 (0.23)  |
|  | bic             | 0.54 (0.02)        | 22.46 (2.35)        | 20.64        | 1.82        | 3.43 (0.2)         | 5.61 (0.3)         | 1.52 (0.05)        | 28.82 (2.45) |
|  | <b>ebic</b>     | <b>0.66</b> (0.02) | 9.52 (1.27)         | 7            | 2.52        | <b>3.63</b> (0.22) | <b>4.53</b> (0.21) | <b>1.53</b> (0.05) | 14.48 (1.35) |
|  | <b>cv</b>       | <u>0.36</u> (0.01) | <u>36.39</u> (1.7)  | <u>35.02</u> | <b>1.37</b> | 3.64 (0.17)        | <u>7.12</u> (0.21) | 1.63 (0.03)        | 43.65 (1.81) |
|  | <b>csuv.m.0</b> | 0.61 (0.02)        | <b>5.79</b> (0.17)  | <b>0.63</b>  | <u>5.16</u> | <u>5.54</u> (0.31) | 5.6 (0.16)         | <u>2.03</u> (0.05) | 5.47 (0.21)  |
|  | <b>csuv.s.0</b> | 0.56 (0.01)        | 10.06 (0.34)        | 6.51         | 3.55        | 4.4 (0.22)         | 5.77 (0.17)        | 1.79 (0.05)        | 12.96 (0.47) |
|  | csuv.m.5        | 0.6 (0.02)         | 5.89 (0.17)         | 0.55         | 5.34        | 5.65 (0.31)        | 5.74 (0.16)        | 2.07 (0.05)        | 5.21 (0.2)   |
|  | csuv.s.5        | 0.55 (0.01)        | 10.51 (0.33)        | 6.95         | 3.56        | 4.49 (0.23)        | 5.91 (0.16)        | 1.81 (0.05)        | 13.39 (0.46) |
|  | csuv.m.0.all    | 0.62 (0.01)        | 5.86 (0.18)         | 0.85         | 5.01        | 5.38 (0.3)         | 5.6 (0.16)         | 2.02 (0.05)        | 5.84 (0.22)  |
|  | csuv.s.0.all    | 0.55 (0.01)        | 11.23 (0.39)        | 7.89         | 3.34        | 4.32 (0.21)        | 5.88 (0.16)        | 1.77 (0.05)        | 14.55 (0.51) |
| csuv.m.0.mcp                                     | 0.46 (0.02)     | 6.98 (0.16)        | 0.05                | 6.93         | 7.18 (0.43) | 6.98 (0.17)        | 2.46 (0.05)        | 3.12 (0.16)        |              |
| csuv.s.0.mcp                                     | 0.58 (0.02)     | 6.33 (0.2)         | 0.9                 | 5.43         | 5.84 (0.34) | 6.06 (0.18)        | 2.16 (0.05)        | 5.47 (0.18)        |              |
| setting 5<br>num.factor = 10<br>s = 5<br>p = 100 | lasso           | 0.39 (0.01)        | 15.08 (0.68)        | 14.51        | 0.57        | 2.5 (0.09)         | 4 (0.16)           | 1.22 (0.03)        | 18.94 (0.71) |
|  | elastic net     | 0.33 (0.01)        | 20.63 (0.96)        | 20.2         | 0.43        | 2.76 (0.1)         | 4.84 (0.21)        | 1.32 (0.03)        | 24.77 (0.97) |
|  | relaxed lasso   | 0.42 (0.01)        | 12.22 (0.73)        | 11.21        | 1.01        | 2.83 (0.13)        | 4.3 (0.21)         | 1.32 (0.04)        | 15.2 (0.8)   |
|  | mcp             | 0.55 (0.02)        | 5.41 (0.29)         | 3.58         | 1.83        | 3.36 (0.21)        | 3.69 (0.2)         | 1.43 (0.07)        | 6.75 (0.21)  |
|  | scad            | 0.51 (0.02)        | 6.87 (0.33)         | 5.25         | 1.62        | 3.59 (0.25)        | 3.64 (0.2)         | 1.42 (0.07)        | 8.63 (0.27)  |
|  | vsd             | 0.57 (0.03)        | 3.84 (0.22)         | 1.02         | 2.82        | 3.96 (0.24)        | 3.65 (0.18)        | 1.59 (0.07)        | 3.2 (0.13)   |
|  | bic             | 0.57 (0.02)        | 5.46 (0.29)         | 3.83         | 1.63        | 3.36 (0.21)        | 3.66 (0.2)         | 1.4 (0.07)         | 7.2 (0.23)   |
|  | <b>ebic</b>     | 0.57 (0.02)        | 5.24 (0.29)         | 3.52         | 1.72        | <u>3.27</u> (0.21) | 3.56 (0.2)         | <u>1.38</u> (0.07) | 6.8 (0.21)   |
|  | <b>cv</b>       | <u>0.39</u> (0.01) | <u>14.96</u> (0.7)  | <u>14.3</u>  | <b>0.66</b> | <b>2.53</b> (0.1)  | <u>4</u> (0.16)    | <b>1.22</b> (0.04) | 18.64 (0.74) |
|  | <b>csuv.m.0</b> | <b>0.63</b> (0.02) | <b>3.61</b> (0.2)   | <b>1.72</b>  | <b>1.89</b> | 2.86 (0.14)        | <b>3.08</b> (0.14) | 1.31 (0.05)        | 4.83 (0.14)  |
|  | <b>csuv.s.0</b> | 0.53 (0.01)        | 6.52 (0.22)         | 5.24         | 1.28        | 2.61 (0.11)        | 3.42 (0.13)        | 1.25 (0.04)        | 8.96 (0.23)  |
|  | csuv.m.5        | 0.62 (0.02)        | 3.48 (0.19)         | 1.48         | 2           | 2.99 (0.15)        | 3.12 (0.15)        | 1.35 (0.05)        | 4.48 (0.13)  |
|  | csuv.s.5        | 0.52 (0.01)        | 6.67 (0.21)         | 5.34         | 1.33        | 2.63 (0.11)        | 3.43 (0.13)        | 1.26 (0.04)        | 9.01 (0.21)  |
|  | csuv.m.0.all    | 0.62 (0.02)        | 3.77 (0.21)         | 1.93         | 1.84        | 2.82 (0.13)        | 3.11 (0.14)        | 1.31 (0.05)        | 5.09 (0.15)  |
|  | csuv.s.0.all    | 0.52 (0.01)        | 7.29 (0.25)         | 6.11         | 1.18        | 2.61 (0.12)        | 3.5 (0.13)         | 1.25 (0.04)        | 9.93 (0.27)  |
| csuv.m.0.mcp                                     | 0.55 (0.02)     | 3.45 (0.16)        | 0.48                | 2.97         | 3.78 (0.21) | 3.43 (0.14)        | 1.59 (0.05)        | 2.51 (0.09)        |              |
| csuv.s.0.mcp                                     | 0.56 (0.02)     | 4.49 (0.21)        | 2.46                | 2.03         | 3.14 (0.15) | 3.49 (0.16)        | 1.42 (0.05)        | 5.43 (0.09)        |              |
| setting 6<br>num.factor = 10<br>s = 5<br>p = 300 | lasso           | 0.28 (0.01)        | 22.46 (1.13)        | 21.37        | 1.09        | 3.19 (0.15)        | 5.23 (0.2)         | 1.46 (0.03)        | 25.28 (1.15) |
|  | elastic net     | 0.23 (0.01)        | 30.03 (1.38)        | 29.08        | 0.95        | 3.48 (0.15)        | 5.98 (0.21)        | 1.55 (0.03)        | 33.13 (1.39) |
|  | relaxed lasso   | 0.32 (0.01)        | 15.45 (0.87)        | 13.71        | 1.74        | 3.55 (0.19)        | 5.2 (0.21)         | 1.56 (0.04)        | 16.97 (0.92) |
|  | mcp             | 0.42 (0.02)        | 7.03 (0.29)         | 4.27         | 2.76        | 4.64 (0.27)        | 4.86 (0.21)        | 1.79 (0.06)        | 6.51 (0.22)  |
|  | scad            | 0.37 (0.02)        | 9.32 (0.3)          | 7.05         | 2.27        | 4.29 (0.29)        | 4.58 (0.22)        | 1.71 (0.07)        | 9.78 (0.25)  |
|  | vsd             | 0.48 (0.02)        | 4.78 (0.18)         | 1.06         | 3.72        | 5.31 (0.3)         | 4.62 (0.16)        | 1.98 (0.06)        | 2.34 (0.12)  |
|  | bic             | 0.41 (0.02)        | 10.11 (1.19)        | 7.7          | 2.41        | 4.57 (0.31)        | 5.2 (0.28)         | 1.77 (0.07)        | 10.29 (1.24) |
|  | <b>ebic</b>     | 0.42 (0.02)        | 7.11 (0.31)         | 4.46         | 2.65        | <u>4.69</u> (0.32) | 4.83 (0.23)        | <u>1.77</u> (0.07) | 6.81 (0.24)  |
|  | <b>cv</b>       | <u>0.29</u> (0.01) | <u>21.97</u> (1.16) | <u>20.83</u> | <b>1.14</b> | <b>3.24</b> (0.16) | <u>5.19</u> (0.21) | <b>1.47</b> (0.03) | 24.69 (1.19) |
|  | <b>csuv.m.0</b> | <b>0.5</b> (0.02)  | <b>4.27</b> (0.16)  | <b>1.35</b>  | <b>2.92</b> | 3.96 (0.2)         | <b>3.85</b> (0.12) | 1.66 (0.04)        | 3.43 (0.12)  |
|  | <b>csuv.s.0</b> | 0.39 (0.01)        | 9.17 (0.28)         | 7.05         | 2.12        | 3.61 (0.18)        | 4.51 (0.14)        | 1.56 (0.04)        | 9.93 (0.29)  |
|  | csuv.m.5        | 0.49 (0.02)        | 4.23 (0.15)         | 1.25         | 2.98        | 4.03 (0.2)         | 3.86 (0.12)        | 1.68 (0.04)        | 3.27 (0.13)  |
|  | csuv.s.5        | 0.39 (0.01)        | 9.12 (0.28)         | 6.97         | 2.15        | 3.68 (0.18)        | 4.57 (0.14)        | 1.58 (0.04)        | 9.82 (0.28)  |
|  | csuv.m.0.all    | 0.5 (0.02)         | 4.42 (0.18)         | 1.67         | 2.75        | 3.82 (0.2)         | 3.81 (0.13)        | 1.61 (0.04)        | 3.92 (0.14)  |
|  | csuv.s.0.all    | 0.37 (0.01)        | 10.27 (0.32)        | 8.26         | 2.01        | 3.52 (0.16)        | 4.63 (0.14)        | 1.55 (0.04)        | 11.25 (0.34) |
| csuv.m.0.mcp                                     | 0.43 (0.01)     | 4.38 (0.12)        | 0.37                | 4.01         | 4.91 (0.26) | 4.23 (0.12)        | 1.93 (0.04)        | 1.36 (0.08)        |              |
| csuv.s.0.mcp                                     | 0.41 (0.02)     | 5.95 (0.17)        | 2.92                | 3.03         | 4.48 (0.24) | 4.63 (0.13)        | 1.8 (0.04)         | 4.89 (0.1)         |              |

Table 9: Model 4: performance of CSUV and methods it compares with (continue). Variable selection performance in terms of F-measure (f), total error (FP+FN), false positives (FP) and false negatives (FN), prediction error in terms of mse (pred.err) and estimation error in terms of l1 and l2 distance (l1.diff and l2.diff) and are shown. The numbers are based on 100 simulations. The last 8 rows are the performance of CSUV with different parameters (e.g. csuv.m.0.mcp corresponds to CSUV with MCP as constituent method and  $r = 0$ ). A bold number represents the best result among delete- $n/2$  cross validation, eBIC and CSUV using Lasso, MCP and SCAD while a underlined number represents the worst among them. Standard errors are shown inside the parentheses.

| parameters  | methods   | f                  | FP+FN               | FP           | FN          | pred.err           | l1.diff             | l2.diff            | size         |
|---|---|--------------------|---------------------|--------------|-------------|--------------------|---------------------|--------------------|--------------|
| setting 7<br>num.factor = 10<br>s = 10<br>p = 100 | lasso   | 0.44 (0.01)        | 21.75 (0.68)        | 19.99        | 1.76        | 4.57 (0.2)         | 7.63 (0.19)         | 1.84 (0.04)        | 28.23 (0.77) |
|   | elastic net                                       | 0.4 (0.01)         | 27.39 (0.79)        | 26.07        | 1.32        | 4.83 (0.23)        | 8.52 (0.24)         | 1.89 (0.04)        | 34.75 (0.86) |
|   | relaxed lasso                                     | 0.46 (0.01)        | 19.07 (0.88)        | 16.61        | 2.46        | 5.34 (0.26)        | 8.24 (0.27)         | 2 (0.04)           | 24.15 (1.05) |
|   | mcp   | 0.51 (0.02)        | 9.26 (0.28)         | 4.25         | 5.01        | 7.58 (0.44)        | 8.19 (0.24)         | 2.47 (0.06)        | 9.24 (0.27)  |
|   | scad  | 0.51 (0.02)        | 9.76 (0.32)         | 5.05         | 4.71        | 7.41 (0.43)        | 7.93 (0.25)         | 2.43 (0.06)        | 10.34 (0.22) |
|   | vsd   | 0.42 (0.02)        | 8.28 (0.23)         | 1.28         | 7           | 8.89 (0.52)        | 8.34 (0.23)         | 2.7 (0.06)         | 4.28 (0.18)  |
|   | bic   | 0.55 (0.02)        | 8.96 (0.39)         | 4.57         | 4.39        | 6.89 (0.41)        | 7.68 (0.25)         | 2.32 (0.06)        | 10.18 (0.38) |
|   | <b>ebic</b>                                       | 0.55 (0.02)        | 8.62 (0.3)          | 3.98         | 4.64        | <u>7</u> (0.41)    | <u>7.7</u> (0.25)   | <u>2.36</u> (0.06) | 9.34 (0.25)  |
|   | <b>cv</b>   | <u>0.44</u> (0.01) | <u>21.75</u> (0.68) | <u>19.99</u> | <b>1.76</b> | <b>4.57</b> (0.2)  | 7.63 (0.19)         | <b>1.84</b> (0.04) | 28.23 (0.77) |
|   | <b>csuv.m.0</b>                                   | <b>0.58</b> (0.01) | <b>7.1</b> (0.2)    | <b>2.14</b>  | <u>4.96</u> | 6.13 (0.36)        | <b>6.78</b> (0.17)  | 2.19 (0.04)        | 7.18 (0.2)   |
|   | <b>csuv.s.0</b>                                   | 0.53 (0.01)        | 10.88 (0.25)        | 7.16         | 3.72        | 5.34 (0.24)        | 7.08 (0.18)         | 2.05 (0.04)        | 13.44 (0.32) |
|   | csuv.m.5  | 0.57 (0.01)        | 7.11 (0.2)          | 1.98         | 5.13        | 6.45 (0.38)        | 6.84 (0.17)         | 2.22 (0.04)        | 6.85 (0.21)  |
|   | csuv.s.5  | 0.53 (0.01)        | 10.87 (0.25)        | 7.15         | 3.72        | 5.25 (0.22)        | 7.05 (0.18)         | 2.04 (0.04)        | 13.43 (0.32) |
|   | csuv.m.0.all                                      | 0.58 (0.01)        | 7.41 (0.21)         | 2.66         | 4.75        | 5.94 (0.32)        | 6.78 (0.18)         | 2.16 (0.05)        | 7.91 (0.22)  |
|   | csuv.s.0.all                                      | 0.52 (0.01)        | 11.72 (0.24)        | 8.26         | 3.46        | 5.27 (0.22)        | 7.18 (0.17)         | 2.03 (0.04)        | 14.8 (0.33)  |
|   | csuv.m.0.mcp                                      | 0.39 (0.01)        | 8.05 (0.16)         | 0.53         | 7.52        | 8.89 (0.47)        | 8.21 (0.17)         | 2.73 (0.05)        | 3.01 (0.12)  |
|   | csuv.s.0.mcp                                      | 0.53 (0.01)        | 7.86 (0.23)         | 2.4          | 5.46        | 7.07 (0.38)        | 7.58 (0.2)          | 2.38 (0.05)        | 6.94 (0.13)  |
|   | setting 8<br>num.factor = 10<br>s = 10<br>p = 300 | lasso              | 0.31 (0.01)         | 32.71 (1.19) | 29.58       | 3.13               | 6.43 (0.26)         | 10.07 (0.22)       | 2.25 (0.03)  |
| elastic net                                       |   | 0.28 (0.01)        | 38.5 (1.24)         | 35.73        | 2.77        | 6.7 (0.26)         | 10.7 (0.19)         | 2.29 (0.03)        | 42.96 (1.35) |
| relaxed lasso                                     |   | 0.32 (0.01)        | 25.3 (1.17)         | 21.11        | 4.19        | 7.22 (0.31)        | 10.41 (0.25)        | 2.39 (0.04)        | 26.92 (1.43) |
| mcp   |   | 0.35 (0.01)        | 12.31 (0.29)        | 5.46         | 6.85        | 9.21 (0.39)        | 10.25 (0.24)        | 2.91 (0.05)        | 8.61 (0.23)  |
| scad  |   | 0.34 (0.01)        | 13.98 (0.35)        | 7.49         | 6.49        | 9.88 (0.46)        | 9.88 (0.25)         | 2.89 (0.06)        | 11 (0.31)    |
| vsd   |   | 0.25 (0.01)        | 10.75 (0.15)        | 1.72         | 9.03        | 12.25 (0.59)       | 10.71 (0.17)        | 3.36 (0.05)        | 2.69 (0.13)  |
| bic   |   | 0.36 (0.02)        | 16.89 (1.3)         | 11.03        | 5.86        | 9.08 (0.44)        | 10.25 (0.28)        | 2.76 (0.06)        | 15.17 (1.52) |
| <b>ebic</b>                                       |   | 0.37 (0.01)        | 12.05 (0.31)        | 5.52         | 6.53        | <u>9.34</u> (0.44) | 9.78 (0.23)         | <u>2.82</u> (0.05) | 8.99 (0.26)  |
| <b>cv</b>   |   | <u>0.31</u> (0.01) | <u>32.71</u> (1.19) | <u>29.58</u> | <b>3.13</b> | <b>6.43</b> (0.26) | <u>10.07</u> (0.22) | <b>2.25</b> (0.03) | 36.45 (1.33) |
| <b>csuv.m.0</b>                                   |   | <b>0.41</b> (0.01) | <b>9</b> (0.2)      | <b>2.15</b>  | <u>6.85</u> | 8.16 (0.35)        | <b>8.41</b> (0.18)  | 2.65 (0.05)        | 5.3 (0.2)    |
| <b>csuv.s.0</b>                                   |   | 0.37 (0.01)        | 15.23 (0.26)        | 9.75         | 5.48        | 7.55 (0.3)         | 9.2 (0.16)          | 2.47 (0.04)        | 14.27 (0.35) |
| csuv.m.5  |   | 0.42 (0.01)        | 8.87 (0.2)          | 1.88         | 6.99        | 8.32 (0.36)        | 8.38 (0.19)         | 2.68 (0.05)        | 4.89 (0.19)  |
| csuv.s.5  |   | 0.37 (0.01)        | 15.25 (0.26)        | 9.77         | 5.48        | 7.41 (0.28)        | 9.2 (0.16)          | 2.47 (0.04)        | 14.29 (0.34) |
| csuv.m.0.all                                      |   | 0.42 (0.01)        | 9.23 (0.22)         | 2.56         | 6.67        | 8.15 (0.34)        | 8.46 (0.18)         | 2.62 (0.04)        | 5.89 (0.2)   |
| csuv.s.0.all                                      |   | 0.36 (0.01)        | 16.64 (0.29)        | 11.47        | 5.17        | 7.39 (0.29)        | 9.41 (0.16)         | 2.45 (0.04)        | 16.3 (0.38)  |
| csuv.m.0.mcp                                      |   | 0.24 (0.01)        | 9.66 (0.13)         | 0.65         | 9.01        | 11.13 (0.5)        | 9.61 (0.16)         | 3.17 (0.05)        | 1.64 (0.09)  |
| csuv.s.0.mcp                                      |   | 0.34 (0.01)        | 10.65 (0.22)        | 3.27         | 7.38        | 9.61 (0.45)        | 9.75 (0.19)         | 2.92 (0.05)        | 5.89 (0.1)   |

Table 10: Model 4: performance of CSUV and methods it compares with (continue). Variable selection performance in terms of F-measure (f), total error (FP+FN), false positives (FP) and false negatives (FN), prediction error in terms of mse (pred.err) and estimation error in terms of l1 and l2 distance (l1.diff and l2.diff) and are shown. The numbers are based on 100 simulations. The last 8 rows are the performance of CSUV with different parameters (e.g. csuv.m.0.mcp corresponds to CSUV with MCP as constituent method and  $r = 0$ ). A bold number represents the best result among delete- $n/2$  cross validation, eBIC and CSUV using Lasso, MCP and SCAD while a underlined number represents the worst among them. Standard errors are shown inside the parentheses.



| parameters                                  | methods         | f                  | FP+FN               | FP           | FN          | pred.err           | l1.diff            | l2.diff            | size         |
|---|-----------------|--------------------|---------------------|--------------|-------------|--------------------|--------------------|--------------------|--------------|
| setting 1<br>rho = 0.5<br>s = 5<br>p = 100  | lasso           | 0.58 (0.02)        | 9.6 (0.72)          | 9.6          | 0           | 1.18 (0.02)        | 1.19 (0.05)        | 0.42 (0.01)        | 14.6 (0.72)  |
|   | elastic net     | 0.49 (0.02)        | 12.7 (0.76)         | 12.7         | 0           | 1.23 (0.02)        | 1.42 (0.06)        | 0.46 (0.01)        | 17.7 (0.76)  |
|   | relaxed lasso   | 0.95 (0.01)        | 0.84 (0.32)         | 0.84         | 0           | 1.08 (0.02)        | 0.67 (0.04)        | 0.32 (0.01)        | 5.84 (0.32)  |
|   | mcp             | 1 (0)              | 0 (0)               | 0            | 0           | 1.06 (0.02)        | 0.6 (0.03)         | 0.33 (0.01)        | 5 (0)        |
|   | scad            | 1 (0)              | 0.01 (0.01)         | 0.01         | 0           | 1.21 (0.03)        | 1.02 (0.05)        | 0.57 (0.03)        | 5.01 (0.01)  |
|   | vsd             | 1 (0)              | 0.03 (0.02)         | 0.03         | 0           | 1.04 (0.02)        | 0.54 (0.02)        | 0.29 (0.01)        | 5.03 (0.02)  |
|   | bic             | 0.92 (0.02)        | 1.41 (0.33)         | 1.41         | 0           | 1.1 (0.02)         | 0.73 (0.04)        | 0.36 (0.02)        | 6.41 (0.33)  |
|   | <b>ebic</b>     | 1 (0)              | <u>0.01</u> (0.01)  | <u>0.01</u>  | 0           | <u>1.07</u> (0.02) | <u>0.62</u> (0.03) | <u>0.33</u> (0.02) | 5.01 (0.01)  |
|   | <b>cv</b>       | 1 (0)              | <b>0</b> (0)        | <b>0</b>     | 0           | 1.06 (0.02)        | 0.6 (0.03)         | <u>0.33</u> (0.01) | 5 (0)        |
|   | <b>csuv.m.0</b> | 1 (0)              | <b>0</b> (0)        | <b>0</b>     | 0           | <b>1.04</b> (0.02) | <b>0.52</b> (0.02) | <b>0.28</b> (0.01) | 5 (0)        |
|   | <b>csuv.s.0</b> | 1 (0)              | <b>0</b> (0)        | <b>0</b>     | 0           | <b>1.04</b> (0.02) | <b>0.52</b> (0.02) | <b>0.28</b> (0.01) | 5 (0)        |
|   | csuv.m.5        | 1 (0)              | 0 (0)               | 0            | 0           | 1.04 (0.02)        | 0.52 (0.02)        | 0.28 (0.01)        | 5 (0)        |
|   | csuv.s.5        | 1 (0)              | 0 (0)               | 0            | 0           | 1.04 (0.02)        | 0.52 (0.02)        | 0.28 (0.01)        | 5 (0)        |
|   | csuv.m.0.all    | 1 (0)              | 0.01 (0.01)         | 0.01         | 0           | 1.04 (0.02)        | 0.53 (0.02)        | 0.28 (0.01)        | 5.01 (0.01)  |
|   | csuv.s.0.all    | 0.99 (0)           | 0.06 (0.02)         | 0.06         | 0           | 1.05 (0.02)        | 0.55 (0.02)        | 0.29 (0.01)        | 5.06 (0.02)  |
|   | csuv.m.0.mcp    | 1 (0)              | 0 (0)               | 0            | 0           | 1.04 (0.02)        | 0.52 (0.02)        | 0.28 (0.01)        | 5 (0)        |
|   | csuv.s.0.mcp    | 1 (0)              | 0 (0)               | 0            | 0           | 1.04 (0.02)        | 0.52 (0.02)        | 0.28 (0.01)        | 5 (0)        |
| setting 2<br>rho = 0.5<br>s = 5<br>p = 300  | lasso           | 0.49 (0.02)        | 14.89 (1.18)        | 14.89        | 0           | 1.23 (0.02)        | 1.46 (0.07)        | 0.45 (0.01)        | 19.89 (1.18) |
|   | elastic net     | 0.39 (0.01)        | 18.8 (1.03)         | 18.8         | 0           | 1.32 (0.02)        | 1.74 (0.06)        | 0.52 (0.01)        | 23.8 (1.03)  |
|   | relaxed lasso   | 0.95 (0.01)        | 0.99 (0.4)          | 0.99         | 0           | 1.08 (0.02)        | 0.65 (0.05)        | 0.31 (0.01)        | 5.99 (0.4)   |
|   | mcp             | 1 (0)              | 0.03 (0.02)         | 0.03         | 0           | 1.06 (0.02)        | 0.57 (0.03)        | 0.3 (0.01)         | 5.03 (0.02)  |
|   | scad            | 0.99 (0)           | 0.07 (0.03)         | 0.07         | 0           | 1.18 (0.02)        | 0.94 (0.05)        | 0.52 (0.03)        | 5.07 (0.03)  |
|   | vsd             | 1 (0)              | 0.01 (0.01)         | 0.01         | 0           | 1.05 (0.02)        | 0.49 (0.02)        | 0.26 (0.01)        | 5.01 (0.01)  |
|   | bic             | 0.6 (0.03)         | 10.86 (1.11)        | 10.86        | 0           | 1.21 (0.02)        | 1.28 (0.07)        | 0.43 (0.02)        | 15.86 (1.11) |
|   | <b>ebic</b>     | <u>0.99</u> (0)    | <u>0.08</u> (0.03)  | <u>0.08</u>  | 0           | <u>1.07</u> (0.02) | <u>0.61</u> (0.04) | <u>0.32</u> (0.02) | 5.08 (0.03)  |
|   | <b>cv</b>       | 1 (0)              | 0.03 (0.02)         | 0.03         | 0           | 1.06 (0.02)        | 0.57 (0.03)        | 0.3 (0.01)         | 5.03 (0.02)  |
|   | <b>csuv.m.0</b> | 1 (0)              | <b>0.01</b> (0.01)  | <b>0.01</b>  | 0           | <b>1.05</b> (0.02) | <b>0.49</b> (0.02) | <b>0.26</b> (0.01) | 5.01 (0.01)  |
|   | <b>csuv.s.0</b> | 1 (0)              | 0.03 (0.02)         | 0.03         | 0           | <b>1.05</b> (0.02) | 0.5 (0.02)         | <b>0.26</b> (0.01) | 5.03 (0.02)  |
|   | csuv.m.5        | 1 (0)              | 0.01 (0.01)         | 0.01         | 0           | 1.05 (0.02)        | 0.49 (0.02)        | 0.26 (0.01)        | 5.01 (0.01)  |
|   | csuv.s.5        | 1 (0)              | 0.01 (0.01)         | 0.01         | 0           | 1.05 (0.02)        | 0.49 (0.02)        | 0.26 (0.01)        | 5.01 (0.01)  |
|   | csuv.m.0.all    | 1 (0)              | 0.03 (0.02)         | 0.03         | 0           | 1.05 (0.02)        | 0.5 (0.02)         | 0.26 (0.01)        | 5.03 (0.02)  |
|   | csuv.s.0.all    | 0.99 (0)           | 0.14 (0.03)         | 0.14         | 0           | 1.07 (0.02)        | 0.55 (0.03)        | 0.28 (0.01)        | 5.14 (0.03)  |
|   | csuv.m.0.mcp    | 1 (0)              | 0 (0)               | 0            | 0           | 1.04 (0.02)        | 0.49 (0.02)        | 0.26 (0.01)        | 5 (0)        |
|   | csuv.s.0.mcp    | 1 (0)              | 0 (0)               | 0            | 0           | 1.04 (0.02)        | 0.49 (0.02)        | 0.26 (0.01)        | 5 (0)        |
| setting 3<br>rho = 0.5<br>s = 10<br>p = 100 | lasso           | 0.65 (0.01)        | 12.17 (0.65)        | 12.17        | 0           | 1.29 (0.02)        | 1.97 (0.06)        | 0.56 (0.01)        | 22.17 (0.65) |
|   | elastic net     | 0.6 (0.01)         | 14.68 (0.71)        | 14.68        | 0           | 1.34 (0.02)        | 2.2 (0.06)         | 0.59 (0.01)        | 24.68 (0.71) |
|   | relaxed lasso   | 0.97 (0.01)        | 0.89 (0.32)         | 0.85         | 0.04        | 1.15 (0.02)        | 1.27 (0.05)        | 0.46 (0.01)        | 10.81 (0.32) |
|   | mcp             | 0.93 (0.01)        | 1.23 (0.09)         | 0.02         | 1.21        | 1.91 (0.05)        | 3.43 (0.1)         | 1.34 (0.04)        | 8.81 (0.09)  |
|   | scad            | 0.95 (0.01)        | 0.87 (0.09)         | 0.12         | 0.75        | 2.41 (0.06)        | 4.35 (0.09)        | 1.65 (0.03)        | 9.37 (0.09)  |
|   | vsd             | 0.99 (0)           | 0.25 (0.05)         | 0            | 0.25        | 1.16 (0.02)        | 1.29 (0.06)        | 0.52 (0.02)        | 9.75 (0.05)  |
|   | bic             | 0.83 (0.02)        | 5.52 (0.73)         | 5.44         | 0.08        | 1.66 (0.05)        | 2.72 (0.11)        | 0.95 (0.05)        | 15.36 (0.74) |
|   | <b>ebic</b>     | 0.94 (0.01)        | 1.24 (0.19)         | 0.71         | <u>0.53</u> | <u>1.95</u> (0.06) | <u>3.37</u> (0.12) | <u>1.29</u> (0.05) | 10.18 (0.22) |
|   | <b>cv</b>       | <u>0.67</u> (0.01) | <u>11.01</u> (0.68) | <u>10.83</u> | 0.18        | 1.37 (0.04)        | 2.15 (0.09)        | 0.65 (0.03)        | 20.65 (0.73) |
|   | <b>csuv.m.0</b> | <b>0.99</b> (0)    | <b>0.14</b> (0.03)  | <b>0.02</b>  | 0.12        | <b>1.13</b> (0.02) | 1.17 (0.04)        | 0.46 (0.02)        | 9.9 (0.04)   |
|   | <b>csuv.s.0</b> | <b>0.99</b> (0)    | 0.21 (0.05)         | 0.15         | <b>0.06</b> | <b>1.13</b> (0.02) | <b>1.15</b> (0.04) | <b>0.44</b> (0.02) | 10.09 (0.04) |
|   | csuv.m.5        | 0.99 (0)           | 0.18 (0.04)         | 0.01         | 0.17        | 1.15 (0.02)        | 1.2 (0.05)         | 0.48 (0.02)        | 9.84 (0.04)  |
|   | csuv.s.5        | 0.99 (0)           | 0.3 (0.06)          | 0.23         | 0.07        | 1.14 (0.02)        | 1.18 (0.04)        | 0.45 (0.02)        | 10.16 (0.05) |
|   | csuv.m.0.all    | 0.99 (0)           | 0.13 (0.03)         | 0.04         | 0.09        | 1.13 (0.02)        | 1.15 (0.04)        | 0.45 (0.02)        | 9.95 (0.04)  |
|   | csuv.s.0.all    | 0.98 (0)           | 0.46 (0.07)         | 0.42         | 0.04        | 1.14 (0.02)        | 1.21 (0.04)        | 0.46 (0.01)        | 10.38 (0.07) |
|   | csuv.m.0.mcp    | 0.91 (0.01)        | 1.62 (0.09)         | 0            | 1.62        | 1.74 (0.05)        | 2.88 (0.13)        | 1.17 (0.05)        | 8.38 (0.09)  |
|   | csuv.s.0.mcp    | 0.89 (0)           | 1.92 (0.05)         | 0            | 1.92        | 1.91 (0.05)        | 3.23 (0.09)        | 1.32 (0.03)        | 8.08 (0.05)  |

Table 11: Model 5: performance of CSUV and methods it compares with. Variable selection performance in terms of F-measure (f), total error (FP+FN), false positives (FP) and false negatives (FN), prediction error in terms of mse (pred.err) and estimation error in terms of l1 and l2 distance (l1.diff and l2.diff) and are shown. The numbers are based on 100 simulations. The last 8 rows are the performance of CSUV with different parameters (e.g. csuv.m.0.mcp corresponds to CSUV with MCP as constituent method and  $r = 0$ ). A bold number represents the best result among delete- $n/2$  cross validation, eBIC and CSUV using Lasso, MCP and SCAD while a underlined number represents the worst among them. Standard errors are shown inside the parentheses.

| parameters                                  | methods         | f                  | FP+FN              | FP           | FN          | pred.err           | l1.diff            | l2.diff            | size         |
|---|-----------------|--------------------|--------------------|--------------|-------------|--------------------|--------------------|--------------------|--------------|
| setting 4<br>rho = 0.5<br>s = 10<br>p = 300 | lasso           | 0.55 (0.01)        | 19.23 (1.1)        | 19.23        | 0           | 1.4 (0.02)         | 2.38 (0.07)        | 0.62 (0.01)        | 29.23 (1.1)  |
|   | elastic net     | 0.48 (0.01)        | 24.02 (1.09)       | 24.02        | 0           | 1.5 (0.03)         | 2.76 (0.07)        | 0.67 (0.01)        | 34.02 (1.09) |
|   | relaxed lasso   | 0.96 (0.01)        | 1.3 (0.44)         | 1.23         | 0.07        | 1.19 (0.02)        | 1.39 (0.06)        | 0.5 (0.02)         | 11.16 (0.44) |
|   | mcp             | 0.92 (0)           | 1.47 (0.09)        | 0.13         | 1.34        | 1.89 (0.04)        | 3.39 (0.09)        | 1.33 (0.03)        | 8.79 (0.08)  |
|   | scad            | 0.93 (0.01)        | 1.31 (0.11)        | 0.51         | 0.8         | 2.41 (0.06)        | 4.29 (0.09)        | 1.62 (0.03)        | 9.71 (0.12)  |
|   | vsd             | 0.96 (0)           | 0.76 (0.07)        | 0.02         | 0.74        | 1.35 (0.03)        | 1.77 (0.08)        | 0.74 (0.03)        | 9.28 (0.07)  |
|   | bic             | 0.61 (0.02)        | 16.37 (1.2)        | 16.32        | 0.05        | 1.49 (0.04)        | 2.54 (0.09)        | 0.72 (0.03)        | 26.27 (1.22) |
|   | <b>ebic</b>     | 0.93 (0.01)        | 1.44 (0.17)        | 0.76         | <u>0.68</u> | <u>2.07</u> (0.06) | <u>3.54</u> (0.11) | <u>1.36</u> (0.04) | 10.08 (0.21) |
|   | <b>cv</b>       | <u>0.56</u> (0.02) | <u>18.5</u> (1.16) | <u>18.38</u> | <b>0.12</b> | 1.44 (0.03)        | 2.49 (0.08)        | 0.67 (0.02)        | 28.26 (1.2)  |
|   | <b>csuv.m.0</b> | <b>0.99</b> (0)    | <b>0.27</b> (0.05) | <b>0.02</b>  | 0.25        | 1.19 (0.02)        | 1.3 (0.06)         | 0.52 (0.02)        | 9.77 (0.05)  |
|   | <b>csuv.s.0</b> | 0.98 (0)           | 0.4 (0.06)         | 0.26         | 0.14        | <b>1.17</b> (0.02) | <b>1.26</b> (0.05) | <b>0.49</b> (0.02) | 10.12 (0.06) |
|   | csuv.m.5        | 0.98 (0)           | 0.33 (0.05)        | 0.02         | 0.31        | 1.21 (0.02)        | 1.34 (0.06)        | 0.54 (0.02)        | 9.71 (0.05)  |
|   | csuv.s.5        | 0.97 (0)           | 0.72 (0.07)        | 0.64         | 0.08        | 1.17 (0.02)        | 1.3 (0.04)         | 0.49 (0.02)        | 10.56 (0.07) |
|   | csuv.m.0.all    | 0.99 (0)           | 0.24 (0.05)        | 0.04         | 0.2         | 1.17 (0.02)        | 1.26 (0.05)        | 0.5 (0.02)         | 9.84 (0.05)  |
|   | csuv.s.0.all    | 0.96 (0)           | 0.8 (0.08)         | 0.72         | 0.08        | 1.19 (0.02)        | 1.32 (0.05)        | 0.5 (0.02)         | 10.64 (0.07) |
|   | csuv.m.0.mcp    | 0.88 (0.01)        | 2.1 (0.08)         | 0            | 2.1         | 2.07 (0.08)        | 3.49 (0.13)        | 1.4 (0.05)         | 7.9 (0.08)   |
| csuv.s.0.mcp                                | 0.89 (0)        | 2.01 (0.05)        | 0                  | 2.01         | 1.99 (0.05) | 3.36 (0.1)         | 1.37 (0.03)        | 7.99 (0.05)        |              |
| setting 5<br>rho = -0.5<br>s = 5<br>p = 100 | lasso           | 0.3 (0.01)         | 25.34 (0.85)       | 25.34        | 0           | 1.46 (0.03)        | 3.36 (0.09)        | 0.92 (0.02)        | 30.34 (0.85) |
|   | elastic net     | 0.23 (0)           | 34.05 (0.8)        | 34.05        | 0           | 1.69 (0.04)        | 4.51 (0.1)         | 1.1 (0.02)         | 39.05 (0.8)  |
|   | relaxed lasso   | 0.62 (0.01)        | 7.04 (0.48)        | 7.04         | 0           | 1.27 (0.03)        | 1.76 (0.09)        | 0.57 (0.02)        | 12.04 (0.48) |
|   | mcp             | 0.98 (0.01)        | 0.26 (0.06)        | 0.26         | 0           | 1.05 (0.02)        | 0.55 (0.02)        | 0.28 (0.01)        | 5.26 (0.06)  |
|   | scad            | 0.96 (0.01)        | 0.55 (0.11)        | 0.55         | 0           | 1.05 (0.02)        | 0.55 (0.02)        | 0.28 (0.01)        | 5.55 (0.11)  |
|   | vsd             | 1 (0)              | 0.01 (0.01)        | 0.01         | 0           | 1.05 (0.02)        | 0.53 (0.02)        | 0.28 (0.01)        | 5.01 (0.01)  |
|   | bic             | 0.95 (0.01)        | 0.6 (0.11)         | 0.6          | 0           | 1.05 (0.02)        | 0.56 (0.02)        | 0.28 (0.01)        | 5.6 (0.11)   |
|   | <b>ebic</b>     | <u>0.96</u> (0.01) | <u>0.43</u> (0.09) | <u>0.43</u>  | 0           | <b>1.05</b> (0.02) | 0.55 (0.02)        | <b>0.28</b> (0.01) | 5.43 (0.09)  |
|   | <b>cv</b>       | 0.97 (0.01)        | 0.41 (0.09)        | 0.41         | 0           | <b>1.05</b> (0.02) | 0.55 (0.02)        | <b>0.28</b> (0.01) | 5.41 (0.09)  |
|   | <b>csuv.m.0</b> | <b>1</b> (0)       | <b>0.03</b> (0.02) | <b>0.03</b>  | 0           | <b>1.05</b> (0.02) | <b>0.54</b> (0.02) | <b>0.28</b> (0.01) | 5.03 (0.02)  |
|   | <b>csuv.s.0</b> | 0.98 (0)           | 0.24 (0.05)        | 0.24         | 0           | <u>1.07</u> (0.02) | <u>0.61</u> (0.03) | <u>0.31</u> (0.01) | 5.24 (0.05)  |
|   | csuv.m.5        | 1 (0)              | 0.03 (0.02)        | 0.03         | 0           | 1.05 (0.02)        | 0.54 (0.02)        | 0.28 (0.01)        | 5.03 (0.02)  |
|   | csuv.s.5        | 0.94 (0.01)        | 0.68 (0.07)        | 0.68         | 0           | 1.1 (0.02)         | 0.74 (0.03)        | 0.36 (0.01)        | 5.68 (0.07)  |
|   | csuv.m.0.all    | 1 (0)              | 0.03 (0.02)        | 0.03         | 0           | 1.05 (0.02)        | 0.54 (0.02)        | 0.28 (0.01)        | 5.03 (0.02)  |
|   | csuv.s.0.all    | 0.98 (0)           | 0.27 (0.05)        | 0.27         | 0           | 1.07 (0.02)        | 0.62 (0.03)        | 0.32 (0.01)        | 5.27 (0.05)  |
|   | csuv.m.0.mcp    | 1 (0)              | 0.02 (0.01)        | 0.02         | 0           | 1.05 (0.02)        | 0.53 (0.02)        | 0.28 (0.01)        | 5.02 (0.01)  |
| csuv.s.0.mcp                                | 1 (0)           | 0.03 (0.02)        | 0.03               | 0            | 1.05 (0.02) | 0.54 (0.02)        | 0.28 (0.01)        | 5.03 (0.02)        |              |
| setting 6<br>rho = -0.5<br>s = 5<br>p = 300 | lasso           | 0.21 (0)           | 40.76 (1.16)       | 40.76        | 0           | 1.86 (0.05)        | 5.06 (0.14)        | 1.28 (0.03)        | 45.76 (1.16) |
|   | elastic net     | 0.15 (0)           | 56.91 (1.19)       | 56.9         | 0.01        | 2.72 (0.08)        | 7.8 (0.19)         | 1.79 (0.04)        | 61.89 (1.19) |
|   | relaxed lasso   | 0.41 (0.01)        | 17.7 (1.17)        | 17.64        | 0.06        | 1.74 (0.06)        | 3.86 (0.2)         | 0.98 (0.04)        | 22.58 (1.18) |
|   | mcp             | 0.96 (0.01)        | 0.48 (0.1)         | 0.48         | 0           | 1.05 (0.02)        | 0.56 (0.03)        | 0.28 (0.01)        | 5.48 (0.1)   |
|   | scad            | 0.92 (0.01)        | 1.09 (0.16)        | 1.09         | 0           | 1.05 (0.02)        | 0.56 (0.02)        | 0.28 (0.01)        | 6.09 (0.16)  |
|   | vsd             | 1 (0)              | 0.04 (0.02)        | 0.04         | 0           | 1.05 (0.02)        | 0.53 (0.02)        | 0.28 (0.01)        | 5.04 (0.02)  |
|   | bic             | 0.82 (0.03)        | 7.95 (1.89)        | 7.95         | 0           | 1.19 (0.05)        | 1.31 (0.21)        | 0.41 (0.04)        | 12.95 (1.89) |
|   | <b>ebic</b>     | 0.93 (0.01)        | 0.81 (0.11)        | 0.81         | <b>0</b>    | <b>1.05</b> (0.02) | <b>0.56</b> (0.03) | <b>0.28</b> (0.01) | 5.81 (0.11)  |
|   | <b>cv</b>       | <u>0.92</u> (0.01) | <u>1.04</u> (0.16) | <u>1.04</u>  | <b>0</b>    | <b>1.05</b> (0.02) | <b>0.56</b> (0.02) | <b>0.28</b> (0.01) | 6.04 (0.16)  |
|   | <b>csuv.m.0</b> | <b>1</b> (0)       | <b>0.05</b> (0.02) | <b>0.04</b>  | <u>0.01</u> | 1.07 (0.02)        | <b>0.56</b> (0.04) | 0.3 (0.02)         | 5.03 (0.02)  |
|   | <b>csuv.s.0</b> | 0.96 (0.01)        | 0.51 (0.08)        | 0.51         | <b>0</b>    | <u>1.1</u> (0.02)  | <u>0.7</u> (0.04)  | <u>0.35</u> (0.01) | 5.51 (0.08)  |
|   | csuv.m.5        | 0.99 (0)           | 0.06 (0.02)        | 0.05         | 0.01        | 1.07 (0.02)        | 0.57 (0.04)        | 0.3 (0.02)         | 5.04 (0.02)  |
|   | csuv.s.5        | 0.86 (0.01)        | 1.73 (0.11)        | 1.73         | 0           | 1.18 (0.02)        | 1.03 (0.04)        | 0.46 (0.01)        | 6.73 (0.11)  |
|   | csuv.m.0.all    | 1 (0)              | 0.05 (0.02)        | 0.04         | 0.01        | 1.07 (0.02)        | 0.56 (0.04)        | 0.3 (0.02)         | 5.03 (0.02)  |
|   | csuv.s.0.all    | 0.95 (0.01)        | 0.54 (0.08)        | 0.54         | 0           | 1.1 (0.02)         | 0.71 (0.04)        | 0.35 (0.02)        | 5.54 (0.08)  |
|   | csuv.m.0.mcp    | 0.99 (0)           | 0.05 (0.03)        | 0.02         | 0.03        | 1.08 (0.03)        | 0.59 (0.05)        | 0.31 (0.03)        | 4.99 (0.03)  |
| csuv.s.0.mcp                                | 0.99 (0)        | 0.12 (0.04)        | 0.12               | 0            | 1.06 (0.02) | 0.56 (0.03)        | 0.29 (0.01)        | 5.12 (0.04)        |              |

Table 12: Model 5: performance of CSUV and methods it compares with (continue). Variable selection performance in terms of F-measure (f), total error (FP+FN), false positives (FP) and false negatives (FN), prediction error in terms of mse (pred.err) and estimation error in terms of l1 and l2 distance (l1.diff and l2.diff) and are shown. The numbers are based on 100 simulations. The last 8 rows are the performance of CSUV with different parameters (e.g. csuv.m.0.mcp corresponds to CSUV with MCP as constituent method and  $r = 0$ ). A bold number represents the best result among delete- $n/2$  cross validation, eBIC and CSUV using Lasso, MCP and SCAD while a underlined number represents the worst among them. Standard errors are shown inside the parentheses.

| parameters                                   | methods         | f                  | FP+FN              | FP          | FN          | pred.err           | l1.diff            | l2.diff            | size         |
|--|-----------------|--------------------|--------------------|-------------|-------------|--------------------|--------------------|--------------------|--------------|
| setting 7<br>rho = -0.5<br>s = 10<br>p = 100 | lasso           | 0.37 (0)           | 34.06 (0.68)       | 33.96       | 0.1         | 1.86 (0.04)        | 5.83 (0.12)        | 1.24 (0.03)        | 43.86 (0.7)  |
|  | elastic net     | 0.35 (0)           | 38.37 (0.66)       | 38.29       | 0.08        | 2.09 (0.05)        | 6.83 (0.13)        | 1.4 (0.03)         | 48.21 (0.68) |
|  | relaxed lasso   | 0.48 (0.01)        | 22.32 (1.04)       | 21.73       | 0.59        | 1.87 (0.04)        | 5.2 (0.14)         | 1.18 (0.03)        | 31.14 (1.15) |
|  | mcp             | 0.85 (0.01)        | 3.04 (0.21)        | 1.53        | 1.51        | 1.56 (0.04)        | 2.69 (0.12)        | 1.06 (0.04)        | 10.02 (0.15) |
|  | scad            | 0.76 (0.01)        | 5.14 (0.26)        | 3.25        | 1.89        | 1.87 (0.05)        | 3.81 (0.11)        | 1.45 (0.03)        | 11.36 (0.23) |
|  | vsd             | 0.81 (0.01)        | 3.01 (0.19)        | 0.16        | 2.85        | 1.81 (0.06)        | 3.24 (0.14)        | 1.3 (0.05)         | 7.31 (0.18)  |
|  | bic             | 0.84 (0.01)        | 3.58 (0.46)        | 2.21        | 1.37        | 1.58 (0.04)        | 2.78 (0.13)        | 1.08 (0.04)        | 10.84 (0.46) |
|  | <b>ebic</b>     | <b>0.85</b> (0.01) | <b>3.09</b> (0.21) | 1.59        | <b>1.5</b>  | <b>1.58</b> (0.04) | <b>2.74</b> (0.12) | <b>1.07</b> (0.05) | 10.09 (0.16) |
|  | <b>cv</b>       | 0.77 (0.01)        | <u>5.01</u> (0.27) | <u>3.09</u> | 1.92        | 1.83 (0.05)        | 3.68 (0.12)        | 1.41 (0.04)        | 11.17 (0.24) |
|  | <b>csuv.m.0</b> | <u>0.76</u> (0.01) | 3.92 (0.13)        | <b>0.22</b> | <u>3.7</u>  | <u>2.08</u> (0.05) | <u>3.95</u> (0.11) | <u>1.57</u> (0.04) | 6.52 (0.13)  |
|  | <b>csuv.s.0</b> | 0.8 (0.01)         | 3.83 (0.18)        | 1.56        | 2.27        | 1.85 (0.06)        | 3.44 (0.13)        | 1.3 (0.04)         | 9.29 (0.19)  |
|  | csuv.m.5        | 0.75 (0.01)        | 3.99 (0.13)        | 0.17        | 3.82        | 2.11 (0.05)        | 4.03 (0.11)        | 1.6 (0.03)         | 6.35 (0.13)  |
|  | csuv.s.5        | 0.79 (0.01)        | 4.15 (0.18)        | 1.89        | 2.26        | 1.92 (0.06)        | 3.59 (0.13)        | 1.33 (0.04)        | 9.63 (0.17)  |
|  | csuv.m.0.all    | 0.76 (0.01)        | 3.87 (0.13)        | 0.25        | 3.62        | 2.06 (0.05)        | 3.91 (0.11)        | 1.55 (0.03)        | 6.63 (0.13)  |
|  | csuv.s.0.all    | 0.8 (0.01)         | 3.86 (0.17)        | 1.71        | 2.15        | 1.84 (0.06)        | 3.4 (0.12)         | 1.28 (0.04)        | 9.56 (0.18)  |
|  | csuv.m.0.mcp    | 0.72 (0.01)        | 4.25 (0.12)        | 0.03        | 4.22        | 2.17 (0.05)        | 4.22 (0.11)        | 1.67 (0.03)        | 5.81 (0.12)  |
| csuv.s.0.mcp                                 | 0.79 (0.01)     | 3.55 (0.16)        | 0.54               | 3.01        | 1.97 (0.06) | 3.67 (0.12)        | 1.44 (0.04)        | 7.53 (0.14)        |              |
| setting 8<br>rho = -0.5<br>s = 10<br>p = 300 | lasso           | 0.26 (0)           | 50.38 (1.39)       | 48.77       | 1.61        | 3.4 (0.11)         | 10.41 (0.21)       | 2.23 (0.05)        | 57.16 (1.54) |
|  | elastic net     | 0.22 (0)           | 59.24 (1.43)       | 57.32       | 1.92        | 4.64 (0.17)        | 13.28 (0.23)       | 2.75 (0.06)        | 65.4 (1.63)  |
|  | relaxed lasso   | 0.35 (0.01)        | 26.61 (1.59)       | 22.95       | 3.66        | 3.76 (0.14)        | 9.65 (0.24)        | 2.33 (0.06)        | 29.29 (1.96) |
|  | mcp             | 0.76 (0.01)        | 5.25 (0.28)        | 3.29        | 1.96        | 1.68 (0.05)        | 3.11 (0.13)        | 1.18 (0.05)        | 11.33 (0.25) |
|  | scad            | 0.63 (0.01)        | 9.48 (0.37)        | 7.35        | 2.13        | 1.97 (0.05)        | 4.2 (0.12)         | 1.5 (0.04)         | 15.22 (0.33) |
|  | vsd             | 0.67 (0.01)        | 4.87 (0.16)        | 0.02        | 4.85        | 2.49 (0.07)        | 4.98 (0.16)        | 1.89 (0.05)        | 5.17 (0.16)  |
|  | bic             | 0.61 (0.02)        | 20.31 (2.56)       | 18.77       | 1.54        | 2.04 (0.09)        | 5.1 (0.37)         | 1.4 (0.06)         | 27.23 (2.63) |
|  | <b>ebic</b>     | <b>0.75</b> (0.01) | <b>5.32</b> (0.28) | 3.37        | <b>1.95</b> | <b>1.69</b> (0.05) | <b>3.12</b> (0.13) | <b>1.18</b> (0.05) | 11.42 (0.26) |
|  | <b>cv</b>       | 0.67 (0.01)        | <u>8.23</u> (0.42) | <u>6.13</u> | 2.1         | 1.87 (0.05)        | 3.85 (0.13)        | 1.4 (0.04)         | 14.03 (0.37) |
|  | <b>csuv.m.0</b> | <u>0.59</u> (0.01) | 5.77 (0.11)        | <b>0.09</b> | <u>5.68</u> | <u>2.96</u> (0.08) | <u>5.94</u> (0.14) | <u>2.2</u> (0.04)  | 4.41 (0.12)  |
|  | <b>csuv.s.0</b> | 0.64 (0.01)        | 6.02 (0.17)        | 1.64        | 4.38        | 2.61 (0.09)        | 5.37 (0.16)        | 1.89 (0.05)        | 7.26 (0.23)  |
|  | csuv.m.5        | 0.59 (0.01)        | 5.77 (0.11)        | 0.08        | 5.69        | 2.99 (0.09)        | 5.95 (0.15)        | 2.2 (0.04)         | 4.39 (0.12)  |
|  | csuv.s.5        | 0.64 (0.01)        | 6.69 (0.2)         | 2.65        | 4.04        | 2.56 (0.07)        | 5.41 (0.15)        | 1.84 (0.04)        | 8.61 (0.22)  |
|  | csuv.m.0.all    | 0.59 (0.01)        | 5.77 (0.11)        | 0.09        | 5.68        | 2.96 (0.08)        | 5.94 (0.14)        | 2.2 (0.04)         | 4.41 (0.12)  |
|  | csuv.s.0.all    | 0.64 (0.01)        | 6.01 (0.17)        | 1.66        | 4.35        | 2.61 (0.08)        | 5.37 (0.16)        | 1.89 (0.05)        | 7.31 (0.23)  |
|  | csuv.m.0.mcp    | 0.56 (0.01)        | 6.05 (0.1)         | 0           | 6.05        | 3.18 (0.1)         | 6.32 (0.14)        | 2.32 (0.04)        | 3.95 (0.1)   |
| csuv.s.0.mcp                                 | 0.63 (0.01)     | 5.63 (0.14)        | 0.58               | 5.05        | 2.81 (0.09) | 5.61 (0.15)        | 2.05 (0.05)        | 5.53 (0.17)        |              |

Table 13: Model 5: performance of CSUV and methods it compares with (continue). Variable selection performance in terms of F-measure (f), total error (FP+FN), false positives (FP) and false negatives (FN), prediction error in terms of mse (pred.err) and estimation error in terms of l1 and l2 distance (l1.diff and l2.diff) and are shown. The numbers are based on 100 simulations. The last 8 rows are the performance of CSUV with different parameters (e.g. csuv.m.0.mcp corresponds to CSUV with MCP as constituent method and  $r = 0$ ). A bold number represents the best result among delete- $n/2$  cross validation, eBIC and CSUV using Lasso, MCP and SCAD while a underlined number represents the worst among them. Standard errors are shown inside the parentheses.

| methods         | pred.err            | size         |
|-----------------|---------------------|--------------|
| lasso           | 26.08 (0.39)        | 12.39 (0.09) |
| elastic net     | 26.12 (0.4)         | 12.38 (0.1)  |
| relaxed lasso   | 26.53 (0.42)        | 11.21 (0.14) |
| mcp             | 26.16 (0.39)        | 11.29 (0.15) |
| scad            | 26.1 (0.39)         | 11.54 (0.12) |
| vsd             | 28.54 (0.42)        | 6.03 (0.16)  |
| bic             | 26.2 (0.4)          | 11.05 (0.15) |
| <b>ebic</b>     | 26.19 (0.4)         | 11.07 (0.15) |
| <b>cv</b>       | <b>26.08</b> (0.39) | 12.36 (0.09) |
| <b>csuv.m.0</b> | 26.64 (0.41)        | 10.02 (0.17) |
| <b>csuv.s.0</b> | <u>26.73</u> (0.41) | 9.9 (0.16)   |
| csuv.m.5        | 26.58 (0.41)        | 10.07 (0.16) |
| csuv.s.5        | 26.63 (0.41)        | 10.07 (0.15) |
| csuv.m.0.all    | 26.72 (0.41)        | 9.95 (0.17)  |
| csuv.s.0.all    | 26.76 (0.41)        | 9.81 (0.16)  |
| csuv.m.0.mcp    | 26.33 (0.4)         | 10.52 (0.16) |
| csuv.s.0.mcp    | 26.39 (0.4)         | 10.46 (0.15) |

Table 14: Boston data: performance of CSUV and methods it compares with. The numbers are based on 100 simulations. The last 8 rows are the performance of CSUV with different parameters (e.g. csuv.m.0.mcp corresponds to CSUV with MCP as constituent method and  $r = 0$ ). A bold number represents the best result among delete- $n/2$  cross validation, eBIC and CSUV using Lasso, MCP and SCAD while a underlined number represents the worst among them. Standard errors are shown inside the parentheses.

| methods         | f                  | FP+FN              | FP           | FN          | size         |
|-----------------|--------------------|--------------------|--------------|-------------|--------------|
| lasso           | 0.46 (0.01)        | 19.96 (1.45)       | 16.88        | 3.08        | 23.8 (1.48)  |
| elastic net     | 0.45 (0.01)        | 22.59 (1.51)       | 20.44        | 2.15        | 28.29 (1.53) |
| relaxed lasso   | 0.56 (0.02)        | 10.15 (0.81)       | 5.85         | 4.3         | 11.55 (0.97) |
| mcp             | 0.45 (0.01)        | 8.97 (0.22)        | 2.62         | 6.35        | 6.27 (0.25)  |
| scad            | 0.48 (0.02)        | 13.01 (0.67)       | 8.35         | 4.66        | 13.69 (0.56) |
| vsd             | NaN (NA)           | 10 (0)             | 0            | 10          | 0 (0)        |
| bic             | 0.43 (0.01)        | 15.78 (1.47)       | 10.5         | 5.28        | 15.22 (1.65) |
| <b>ebic</b>     | <u>0.44</u> (0.01) | 11.33 (1.18)       | 5.09         | 6.24        | 8.85 (1.31)  |
| <b>cv</b>       | 0.45 (0.02)        | <u>19.3</u> (1.45) | <u>15.82</u> | <b>3.48</b> | 22.34 (1.54) |
| <b>csuv.m.0</b> | 0.45 (0.01)        | 7.04 (0.12)        | <b>0.03</b>  | <u>7.01</u> | 3.02 (0.12)  |
| <b>csuv.s.0</b> | <b>0.65</b> (0.01) | <b>5.49</b> (0.19) | 0.99         | 4.5         | 6.49 (0.32)  |
| csuv.m.5        | 0.43 (0.01)        | 7.26 (0.12)        | 0.02         | 7.24        | 2.78 (0.12)  |
| csuv.s.5        | 0.69 (0.01)        | 5.07 (0.19)        | 1.08         | 3.99        | 7.09 (0.3)   |
| csuv.m.0.all    | 0.49 (0.01)        | 6.72 (0.13)        | 0.02         | 6.7         | 3.32 (0.13)  |
| csuv.s.0.all    | 0.69 (0.01)        | 5.16 (0.22)        | 1.3          | 3.86        | 7.44 (0.35)  |
| csuv.m.0.mcp    | 0.28 (0.01)        | 8.61 (0.09)        | 0            | 8.61        | 1.39 (0.09)  |
| csuv.s.0.mcp    | 0.44 (0.01)        | 7.19 (0.09)        | 0.05         | 7.14        | 2.91 (0.09)  |

Table 15: Riboflavin data with permutation: performance of CSUV and methods it compares with. The numbers are based on 100 simulations. The last 8 rows are the performance of CSUV with different parameters (e.g. csuv.m.0.mcp corresponds to CSUV with MCP as constituent method and  $r = 0$ ). A bold number represents the best result among delete- $n/2$  cross validation, eBIC and CSUV using Lasso, MCP and SCAD while a underlined number represents the worst among them. Standard errors are shown inside the parentheses.