Simultaneous change-point and factor analysis for high-dimensional time series

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High-dimensional factor models

- Popular dimension reduction technique for high-dimensional time series in finance, economics, neuroscience, biology, seismology.
- A small number of unobserved factors drive the co-movement of a large number of time series:

\[
x_{it} = \chi_{it}^\top \lambda_f + \epsilon_{it} = \sum_{i=1}^{r} \lambda_{ij} f_{jt} + \epsilon_{it}, \quad 1 \leq i \leq n; \quad 1 \leq t \leq T.
\]

- Typically assume that the factor loadings and factors are time-invariant.
- Use principal component analysis (PCA) to estimate the factor space.
- Successfully applied to business cycle analysis, consumer behaviour analysis, asset pricing and economic monitoring and forecasting.
Change-points in high-dimensional time series

- During financial crisis, stronger co-movements are observed ⇒ fewer factors capture larger shares of total variance ⇒ change in the number of factors and factor structure in general.
- $x_{it}$ = log returns of daily closing values of stocks composing S&P100 index (04/01/2000 – 10/08/2016, $n = 88$ and $T = 4177$).

**Vertical broken lines**: change-points detected from the common components.
Minimum number of eigenvalues required to account for 50–95% (y-axis) of the variance of $x_t$. 

Change-points in high-dimensional time series
Change-points in the common components

Standard factor modelling methods are ‘fooled’ by structural breaks.

\[
\chi_{it} = \begin{cases} 
    a_i g_{1t} = a_i (\alpha g_{1,t-1} + u_t) & \text{for } \eta_0^x + 1 = 1 \leq t \leq \eta_1^x, \\
    a_i g_{1t} + b_i g_{2t} = a_i (\alpha g_{1,t-1} + u_t) + b_i g_{2t} & \text{for } \eta_1^x + 1 \leq t \leq \eta_2^x, \\
    c_i g_{1t} + d_i g_{2t} = c_i (\alpha g_{1,t-1} + u_t) + d_i g_{2t} & \text{for } \eta_2^x + 1 \leq t \leq \eta_3^x, \\
    c_i g_{1t} + d_i g_{2t} = c_i (\beta g_{1,t-1} + u_t) + d_i g_{2t} & \text{for } \eta_3^x + 1 \leq t \leq \eta_4^x = T,
\end{cases}
\]

= time-invariant loadings \( \lambda_i \) \times time-varying factors \( f_t \)

\[
= \begin{bmatrix} a_i & b_i & c_i & d_i \end{bmatrix}_{r=4} \times \begin{bmatrix} (\alpha g_{1,t-1} + u_t, 0, 0, 0)^\top \\
(\alpha g_{1,t-1} + u_t, g_{2t}, 0, 0)^\top \\
(0, 0, \alpha g_{1,t-1} + u_t, g_{2t})^\top \\
(0, 0, \beta g_{1,t-1} + u_t, g_{2t})^\top \end{bmatrix}
\]

\[
\text{for } 1 \leq t \leq \eta_1^x, \\
\text{for } \eta_1^x + 1 \leq t \leq \eta_2^x, \\
\text{for } \eta_2^x + 1 \leq t \leq \eta_3^x, \\
\text{for } \eta_3^x + 1 \leq t \leq T.
\]

\( \therefore \) instabilities in loadings and factor number are absorbed into instabilities in the second-order structure of the factors with \( r = 4 \).
Literature review

Importance of change-point analysis has increasingly been noted in many applications (e.g., forecasting with factor-augmented models).

Most of existing change-point methods for factor models test

\[ H_0 : \text{loadings are stable. vs } H_1 : \text{there exists a single break in the loadings.} \]

• Investigation is devoted to the effect of a single break in the loadings or factor number on estimation, with accompanying change-point tests.

Lasso-type estimators: Cheng et al. (2016) for detecting and locating a single change-point, and Ma & Su (2016) for multiple change-point detection in the loadings.

So far, the existing literature overwhelmingly focuses on single change-point detection, excluding change-points in the autocorrelations of factors and those in the idiosyncratic components.
In this talk

(a) Propose a comprehensive methodology for **consistent estimation of multiple change-points**, in the second-order structure of a high-dimensional time series governed by a factor model.

- Advocate the marriage of factor analysis and change-point analysis.

(b) Operate under the **most flexible piecewise-stationarity**, embracing **all possible structural instabilities** under factor modelling.

(c) Through the use of wavelets, change-point detection in the **second-order** structure of a high-dimensional time series $\Rightarrow$ change-point detection in the **means** of high-dimensional panel data.

(d) Circumvent the difficult problem of accurate estimation of the true factor number by adopting a **screening** procedure.
**Piecewise stationary factor models**

\[ x_{it} = \chi_{it} + \epsilon_{it} = \lambda_i^\top f_t + \epsilon_{it}, \quad 1 \leq i \leq n; \quad 1 \leq t \leq T. \]

- \( f_t = (f_{1t}, \ldots, f_{rt})^\top \): \( r \) factors driving the common components.
- \( \lambda_i = (\lambda_{i1}, \ldots, \lambda_{ir})^\top \): time-invariant loadings.

† **loadings with breaks** × factors
  = time-invariant loadings × **piecewise stationary factors**

\( (A1.\chi) \) **Change-points in common components**: \( B^\chi = \{\eta_{1\chi}, \ldots, \eta_{B\chi}\} \).

(i) For each \( b = 0, \ldots, B_\chi \), \( \exists \) weakly stationary processes \( f_t^b \) such that

\[ \mathbb{E}\|f_t - f_t^b\|^2 \to 0 \text{ as } t - \eta_b^\chi \to \infty. \]

(ii) Let \( \chi_{it}^b = \lambda_i^\top f_t^b \). There exists \( \bar{\tau}_\chi < \infty \) such that for all \( b \),

\[ \text{ACV of } \chi_t^b - \text{ACV of } \chi_t^{b+1} \neq O_n \text{ at some lag } \leq \bar{\tau}_\chi. \]

\( (A1.\epsilon) \) **Change-points in idiosyncratic components**: \( B^\epsilon = \{\eta_{1\epsilon}, \ldots, \eta_{B\epsilon}\} \).
(A2) \( E(\chi_{it}\epsilon_{i't'}) = 0 \) for any \( i, i' = 1, \ldots, n \) and \( t, t' = 1, \ldots, T \).

(A3) There exists \( C_f < \infty \) such that \( \|E(ftf_t^\top)\| < C_f \).

(A4) \( n^{-1}\Lambda^\top\Lambda \to H \) as \( n \to \infty \) for positive definite \( H \). Also, \( \max_{i,j} |\lambda_{ij}| < \lambda < \infty \).

(A5) \( \exists C_\epsilon < \infty \) such that, for any \( \{a_i\}_{i=1}^n \) satisfying \( \sum_{i=1}^n a_i^2 = 1 \),

\[
\sup_{I, I' \subseteq \{1, \ldots, T\}} \frac{1}{\sqrt{|I||I'|}} \sum_{i=1}^n \sum_{i'=1}^n \sum_{t \in I} \sum_{t' \in I'} a_i a_i' E(\epsilon_{it}\epsilon_{i't'}) < C_\epsilon.
\]

(A6) \( (\min_{0 \leq b \leq B_\chi} |\eta_{b+1}^\chi - \eta_b^\chi| \wedge \min_{0 \leq b \leq B_\epsilon} |\eta_{b+1}^\epsilon - \eta_b^\epsilon|) \geq cT \) for some \( c \in (0, 1) \).

Under (A2)–(A6), \( \Gamma_x = T^{-1} \sum_{t=1}^T E(x_t x_t^\top) \):

(i) its \( r \) largest eigenvalues are diverging linearly in \( n \) as \( n \to \infty \);  

(ii) the \( (r + 1) \)th largest eigenvalue stays bounded for any \( n \).

\( \therefore \) For identification of \( \chi_{it} \) and \( \epsilon_{it} \), we operate in an asymptotic setting where

(A7) \( n \to \infty \) as \( T \to \infty \) while \( n/T = O(\log^2 T) \).
Consistency of PCA

• $\hat{\Gamma}_x = T^{-1} \sum_{t=1}^{T} x_t x_t^\top$: sample covariance matrix.
• $\hat{w}_{x,j}$: normalised eigenvector of $\hat{\Gamma}_x$ corresponding to its $j$th largest eigenvalue.

PCA estimators of $\chi_{it}$ and $\epsilon_{it}$ for $k$ as a factor number:

$$\hat{\chi}_t^k = \sum_{j=1}^{k} \hat{w}_{x,j} \hat{w}_{x,j}^\top x_t \quad \text{and} \quad \hat{\epsilon}_t^k = x_{it} - \hat{\chi}_t^k.$$ 

When $k = r$: $|\hat{\chi}_t^r - \chi_{it}| = O_p(\frac{1}{\sqrt{T}} \lor \frac{1}{\sqrt{n}})$, as good as under stationary factor models.

When $k \geq r$ and fixed: for any $i$ and $1 \leq s < e \leq T$,

$$\frac{1}{\sqrt{e - s + 1}} \left| \sum_{t=s}^{e} (\hat{\chi}_t^k - \chi_{it}) \right| = O_p \left( \sqrt{\frac{n}{T}} \right) = O_p(\log T).$$

∴ Change-points in the second-order structure of $\chi_{it}$ $(\epsilon_{it})$ appear as those in $\hat{\chi}_t^k$ $(\hat{\epsilon}_t^k)$ for any fixed $k \geq r$. 
Overview of the proposed methodology

Stage 1:
- **PCA**-based estimation of common and idiosyncratic components.
- **Wavelet**-based transformation of the estimated common and idiosyncratic components.

Stage 2: **Double CUSUM Binary Segmentation** (DCBS) algorithm (Cho 2016) for multiple change-point detection from \( \hat{\chi}_{it}^k \) and \( \hat{\epsilon}_{it}^k \), separately.
Stage 1: Wavelet-based transformation

- **Wavelets**: analogous to the Fourier exponentials in the spectral representation for stationary processes, in the **Locally Stationary** model (Nason et al., 2000).
- Filtering $\hat{\chi}_{it}$ with a vector of wavelets $\psi_j = (\psi_{j,0}, \ldots, \psi_{j,L_j-1})^\top$,
  
  wavelet coefficients: $\hat{d}_{j,it} = \sum_l \chi_{i,t-l} \psi_{j,l}$.

- Wavelet-based transformation of $\chi_t$:
  
  $g_j(\chi_{it}) = |\hat{d}_{j,it}| \iff$ within-series structure of $\chi_{it}$,
  
  $h_j(\chi_{it}, \chi_{i't}) = |\hat{d}_{j,it} \pm \hat{d}_{j,i't}| \iff$ cross-series structure of $(\chi_{it}, \chi_{i't})$.

- Change-points in the **second-order structure** of $\chi_t$ are detectable as change-points in the **means** of $g_j(\chi_{it}), 1 \leq i \leq n$ and $h_j(\chi_{it}, \chi_{i't}), 1 \leq i < i' \leq n$ at **multiple** wavelet scales $j$ (Cho & Fryzlewicz, 2015).

- In practice, $\chi_{it}$ is unknown.
  
  - Replace it with $g_j(\hat{\chi}_{it}^k)$ and $h_j(\hat{\chi}_{it}^k, \hat{\chi}_{i't}^k)$. 

Input panel data for segmentation

For some fixed $k \geq r$, consider the $N = J^*_T n(n + 1)/2$-dimensional panel

$$y_{\ell t} = \begin{cases} g_j(\hat{\chi}^k_{it}), & 1 \leq i \leq n, \\ h_j(\hat{\chi}^k_{it}, \hat{\chi}^k_{i't}), & 1 \leq i \leq i' \leq n, \end{cases} \quad \ell = 1, \ldots, N; \ t = 1, \ldots, T,$$

$$= g_{\ell t} + \varepsilon_{\ell t}.$$

(i) $g_{\ell t}$ are piecewise constant with all its change-points $= B^X = \{\eta^X_1, \ldots, \eta^X_{B^X}\}$.

(ii) $\max_{1 \leq \ell \leq N} \max_{1 \leq s < e \leq T} \frac{1}{\sqrt{e-s+1}} | \sum_{t=s}^e \varepsilon_{\ell t} | = O_p(\log^{1+\upsilon} T)$.

$y_{\ell t}$ serves as an input to panel data segmentation algorithm in Stage 2.

† The same arguments hold for $\hat{\varepsilon}^k_{it}$.
Stage 2: Panel data segmentation

**Problem**: detect multiple change-points shared across $g_{\ell t}$

$$y_{\ell t} = g_{\ell t} + \varepsilon_{\ell t}, \quad \ell = 1, \ldots, N; \ t = 1, \ldots, T.$$

CUSUM statistics for **univariate** data: for a given $\ell$,

$$Y^\ell_b = \sqrt{\frac{b(T-b)}{T}} \left( \frac{1}{b} \sum_{t=1}^{b} y_{\ell t} - \frac{1}{T-b} \sum_{t=b+1}^{T} y_{\ell t} \right), \quad b = 1, \ldots, T - 1.$$
Binary segmentation

How do we simultaneously handle the $N$-dimensional CUSUM series from $y_{\ell t}$?
Double CUSUM statistic (Cho 2016)

$$D_b(m) = \sqrt{\frac{m(2N-m)}{2N}} \left( \frac{1}{m} \sum_{\ell=1}^{m} |Y_b^{(\ell)}| - \frac{1}{2N-m} \sum_{\ell=m+1}^{N} |Y_b^{(\ell)}| \right)$$

for all $b = 1, \ldots, T - 1$ and $m = 1, \ldots, N$, where $|Y_b^{(1)}| \geq \ldots \geq |Y_b^{(N)}|$. 

$N = 100, T = 500$, CP at $t = T/2$ with jump size $= 0.15$ for $2\%$ of coordinates.
Double CUSUM statistic (Cho 2016)

\[ \mathcal{D}_b(m) = \sqrt{\frac{m(2N-m)}{2N}} \left( \frac{1}{m} \sum_{\ell=1}^{m} |\mathcal{Y}_b^{(\ell)}| - \frac{1}{2N-m} \sum_{\ell=m+1}^{N} |\mathcal{Y}_b^{(\ell)}| \right) \]

for all \( b = 1, \ldots, T - 1 \) and \( m = 1, \ldots, N \), where \( |\mathcal{Y}_b^{(1)}| \geq \ldots \geq |\mathcal{Y}_b^{(N)}| \).

\[ N = 100, \; T = 500, \; \text{CP at} \; t = T/2 \; \text{with jump size} = 0.15 \; \text{for} \; 2\% \; \text{of coordinates.} \]
Double CUSUM statistic (Cho 2016)

\[ D_b(m) = \sqrt{\frac{m(2N-m)}{2N}} \left( \frac{1}{m} \sum_{\ell=1}^{m} |Y_b^{(\ell)}| - \frac{1}{2N-m} \sum_{\ell=m+1}^{N} |Y_b^{(\ell)}| \right) \]

for all \( b = 1, \ldots, T - 1 \) and \( m = 1, \ldots, N \), where \( |Y_b^{(1)}| \geq \ldots \geq |Y_b^{(N)}| \).

\[ N = 100, T = 500, \text{ CP at } t = T/2 \text{ with jump size } = 0.03 \text{ for } 50\% \text{ of coordinates.} \]
Double CUSUM statistic (Cho 2016)

\[ D_b(m) = \sqrt{\frac{m(2N-m)}{2N}} \left( \frac{1}{m} \sum_{\ell=1}^{m} |Y_b^{(\ell)}| - \frac{1}{2N-m} \sum_{\ell=m+1}^{N} |Y_b^{(\ell)}| \right) \]

for all \( b = 1, \ldots, T - 1 \) and \( m = 1, \ldots, N \), where \( |Y_b^{(1)}| \geq \ldots \geq |Y_b^{(N)}| \).

\[ N = 100, \ T = 500, \ \text{CP at} \ t = T/2 \ \text{with jump size} \ = 0.03 \ \text{for} \ 50\% \ \text{of coordinates}. \]
Double CUSUM Binary Segmentation algorithm

$= \text{DC statistic} + \text{binary segmentation.}$

- $\max_{1 \leq m \leq N} D_b(m)$ replaces CUSUM series and a change-point estimate is
  \[ \hat{\eta} = \arg \max_{b \in [s,e)} \max_{1 \leq m \leq N} D_{s,b,e}(m) \] for a given interval $[s, e]$.

- Majority of the high-dimensional change-point literature concerns the problem of **single** change-point detection.

- DCBS algorithm
  (a) guarantees the consistency in **multiple** change-point detection in high-dimensional settings;
  (b) admits both **serial- and cross-correlations** in the data $\Rightarrow$ highly relevant for the time series factor model.
Consistency of the DCBS algorithm

At each $\eta^\chi_b$, \[ m_b : \text{number of series sharing the change-point, denseness,} \]
\[ \tilde{\delta}_b : \text{average jump size at } \eta^\chi_b, \]
\[ \Delta_{N,T} = \min_{1 \leq b \leq B^\chi} \sqrt{m_b} \tilde{\delta}_b : \text{min. cross-sectional change size}. \]

Suppose \( (\sqrt{N} \log^{1+\nu} T)^{1/4} \Delta_{N,T} T^{1/4} \rightarrow \infty \) (high-dimensional efficiency). Then, for a threshold satisfying \( C' N \Delta_{N,T}^{-1} \log^{2+2\nu} T < \pi_{N,T} < C'' \Delta_{N,T} T^{1/2} \),

\[
P\left( \hat{B}^\chi = B^\chi; \left. \frac{1}{c_1 \omega_{N,T}} \left| \hat{\eta}^\chi_b - \eta^\chi_b \right| \right|_{b = 1, \ldots, \hat{B}^\chi} \rightarrow 1ight)
\]

where $\omega_{N,T} = N \Delta_{N,T}^{-2} \log^{2+2\nu} T$.

† **Near-optimality in change-point detection:** dense and large changes are estimated with bias logarithmic in $T$, close to the optimal bias $O_p(1)$. 

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Bootstrap for threshold selection

- Approximate the quantiles of $T_{s,e}$ under $H_0$ using a resampling scheme based on the factor structure and stationary bootstrap (Politis & Romano, 2004).
- **Stationary Bootstrap**: block bootstrap which generates SB sample $X_{1•}^•, \ldots, X_T^•$ that is stationary conditional on the observed $X_1, \ldots, X_T$, using blocks with lengths independently drawn from a geometric distribution.

**Step 1.** $\chi$: For each $j \in \{1, \ldots, k\}$, produce the SB sample of $\{\hat{f}_{jt}, 1 \leq t \leq T\}$ as $\{f_{jt}^•, 1 \leq t \leq T\}$ and compute $\chi_{it}^k = \sum_{j=1}^k \hat{\lambda}_{ij} f_{jt}^•$.

**Step 1.** $\epsilon$: Produce the SB sample of $\{\hat{\epsilon}_i^k = (\hat{\epsilon}_{1it}, \ldots, \hat{\epsilon}_{nit})^\top, 1 \leq t \leq T\}$ as $\{\epsilon_{it}^k = (\epsilon_{1it}^k, \ldots, \epsilon_{nit}^k)^\top, 1 \leq t \leq T\}$.

**Step 2:** Generate $y_{\ell t}^•$ through transforming $\chi_{it}^k$ or $\epsilon_{it}^k$ using $g_j$ and $h_j$.

**Step 3:** Obtain $Y_{s,b,e}^\ell$ and compute the test statistic $T_{s,e}^•$.

**Step 4:** Repeat Steps 1–3 $R$ times. The critical value $\pi_{s,e}^{\chi}(k)$ or $\pi_{s,e}^{\epsilon}(k)$ for the segment $[s, e]$ is selected as the $(1 - \alpha)$-quantile of the $R$ bootstrap test statistics $T_{s,e}^•$ at a given significance level $\alpha \in (0, 1)$.
Screening over factor number candidates

- Factor number estimation is challenging particularly in the presence of multiple change-points.
- Existing change-point tests for factor models rely on accurately estimating \( r \).
- Our theoretical results hold for over-specified \( k \geq r \), while using \( k < r \) leads to \( \hat{\chi}_it^k \) that may not contain all \( B^x \).
- **Screening** of change-points detected from \( \hat{\chi}_it^k \) for a range of \( k \).
- Factor number candidates: \( \mathcal{R} = \{r, \ldots, \bar{r}\} \)
- Screen \( \hat{B}^x(k) \), a set of estimated change-points from \( \hat{\chi}_it^k \) for all \( k \in \mathcal{R} \), and select \( k \) returning \( \hat{B}^x(k) \) with the largest cardinality:
  \[
  k^* = \max\{k \in \mathcal{R} : |\hat{B}^x(k)| = \max_{k' \in \mathcal{R}} |\hat{B}^x(k')|\}.
  \]
- Change-points in \( \epsilon_{it} \) are detected from \( \hat{\epsilon}_{it}^{k^*} = x_{it} - \hat{\chi}_it^{k^*} \).
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Finite sample performance: single CP scenario

For $i = 1, \ldots, n = 100$ and $t = 1, \ldots, T = 200$,

$$x_{it} = \sum_{j=1}^{q=5} \lambda_{ij} f_{jt} + \sqrt{\theta} \epsilon_{it}, \quad \text{where} \quad \theta = \phi \cdot \frac{q}{1 - \rho_f^2} \cdot \frac{1 - \rho^2}{1 + 2H \beta^2},$$

$$f_{jt} = \rho_{f,j} f_{j,t-1} + u_{jt}, \quad u_{jt} \sim_{iid} \mathcal{N}(0, 1),$$

$$\epsilon_{it} = \rho_{\epsilon,i} \epsilon_{i,t-1} + v_{it} + \beta_i \sum_{|k| \leq H_i, k \neq 0} v_{i+k,t}, \quad v_{it} \sim_{iid} \mathcal{N}(0, 1),$$

- $\rho_{f,j} = 0.4 - 0.05(j - 1)$ and $\rho_{\epsilon,i} \sim_{iid} \mathcal{U}(-0.5, 0.5)$ provide autocorrelations.
- $\beta_i \in \{-0.2, 0.2\}$ and $H = 5$ supply cross-correlations in $\epsilon_t$.
- In each scenario, a single change-point is introduced to loadings, acf of factors, number of factors and the autocorrelations or cross-correlations of idiosyncratic components at $\eta = T/3$, with its denseness controlled by $\varrho \in \{1, 0.75, 0.5, 0.25\}$ (dense to sparse).
- $\phi \in \{1.0, 1.5, 2.0, 2.5\}$ controls signal-to-noise ratio.
- Compare DC ‘test’ (first iteration of DCBS algorithm) against MAX, AVG (pointwise maximum and average of CUSUMs) and DC-NFA (No Factor Analysis, DC test applied to wavelet transformation of $x_{it}$).
Finite sample performance: single CP scenario

A single CP in loadings: detection power of **DC**, **MAX**, **AVG** and **DC-NFA** vs. \( \phi \in \{1.0, 1.5, 2.0, 2.5\} \) for \( \psi \in \{1, 0.75, 0.5, 0.25\} \) (left to right), applied to common (top) and idiosyncratic (bottom) components.
Finite sample performance: single CP scenario

A single CP in acf of factors: detection power of DC, MAX, AVG and DC-NFA vs. \( \phi \in \{1.0, 1.5, 2.0, 2.5\} \), applied to common (top) and idiosyncratic (bottom) components.
Finite sample performance: single CP scenario

A single CP in number of factors: detection power of DC, MAX, AVG and DC-NFA vs. $\phi \in \{1.0, 1.5, 2.0, 2.5\}$ for $\varrho \in \{1, 0.75, 0.5, 0.25\}$ (left to right), applied to common (top) and idiosyncratic (bottom) components.
Finite sample performance: single CP scenario

A single CP in autocorrelations of $\epsilon_{it}$: detection power of DC, MAX, AVG and DC-NFA vs. $\phi^{-1} \in \{2.5, 2.0, 1.5, 1\}$ for $\rho \in \{1, 0.75, 0.5, 0.25\}$ (left to right), applied to common (top) and idiosyncratic (bottom) components.
Finite sample performance: single CP scenario

A single CP in cross-covariance of $\epsilon_{it}$: detection power of DC, MAX, AVG and DC-NFA vs. $\phi^{-1} \in \{2.5, 2.0, 1.5, 1\}$ for $\varrho \in \{1, 0.75, 0.5, 0.25\}$ (left to right), applied to common (top) and idiosyncratic (bottom) components.
Covariance matrix of the idiosyncratic components to the left and right of $\eta = T/3$. 
Application to exchange rates against British Sterling

\( x_{it} = \) log returns of the daily exchange rates of 24 currencies against British Sterling (03/05/2005 – 12/08/2016, \( n = 24 \) and \( T = 2851 \)).

† \( \hat{\eta}_9^x = 15 \) June 2016 (UK’s EU referendum took place on 23 June 2016).
Minimum number of eigenvalues required to account for 50–95% (y-axis) of the variance of $x_t$. 
Conclusions

• New theoretical results on consistency of $\hat{\chi}_{it}^k$ estimated with $k \geq r$.
• Novel two-stage methodology for multiple change-point detection in the factor model with the most flexible change-point structure.
• Consistent detection of multiple change-points in loadings, acf of factors, factor number and idiosyncratic components, as well as identifying their origins.

References


