Supplementary document for "Modelling and forecasting daily electricity load curves: a hybrid approach"

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A Proof of Theorem 1

Based on (9), model (4) can be written as

$$\sum_{k=1}^{\infty} \xi_{ik}\varphi_k(u) = \sum_{k=1}^{\infty} \eta_{ik} \int_{\mathcal{I}_2} \psi_k(v)\beta(u,v)dv + \varepsilon_i(u).$$

Multiplying the both sides of the above equation by $\varphi_j(u)$ and taking the integration with respect to u, we obtain that

$$\xi_{ij} = \sum_{k=1}^{\infty} \beta_{jk}\eta_{ik} + \varepsilon_{ij}, \qquad j = 1, 2, \ldots. \tag{1}$$

For $j > r$, (11) implies that $\mathbb{E}(\xi_{ij}\eta_{ik}) = 0$ for all $k = 1, 2, \ldots$ and thus it follows from (1) that $\text{var}(\xi_{ij}) = \mathbb{E}(\xi_{ij}^2) = \mathbb{E}(\varepsilon_{ij}^2) = \text{var}(\varepsilon_{ij})$ for $j > r$. Since the two terms on the

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RHS of (1) are uncorrelated, it holds that

$$\sum_{k=1}^{\infty} \beta_{jk}\eta_{ik} = 0 \quad \text{a.s.} \quad \text{for all } j > r, \tag{2}$$

and hence (12) holds.

To show that the converse is also true, we first notice that (2) is implied by (11) only and thus still holds. Therefore the first equation in (12) holds effectively for all $j \geq 1$. It follows from (12) and (9) that

$$\begin{aligned} Y_i(u) &= \sum_{j,k=1}^{\infty} \beta_{jk}\eta_{ik}\varphi_j(u) + \sum_{j=1}^{\infty} \varepsilon_{ij}\varphi_j(u) \\ &= \sum_{j,k=1}^{\infty} \varphi_j(u)\eta_{ik} \int_{\mathcal{I}_1\times\mathcal{I}_2} \varphi_j(w)\psi_k(v)\beta(w,v)dwdv + \varepsilon_i(u) \\ &= \sum_{j=1}^{\infty} \varphi_j(u) \int_{\mathcal{I}_1\times\mathcal{I}_2} \varphi_j(w)X_i(v)\beta(w,v)dwdv + \varepsilon_i(u). \end{aligned} \tag{3}$$

The second equality follows from the fact that $\varepsilon_i(u) = \sum_j \varepsilon_{ij}\varphi_j(u)$, since $\{\varphi_j(\cdot)\}$ is an orthonormal basis of $\mathcal{L}_2(\mathcal{I}_1)$. Similarly the third equality holds and combined with (3), (4) immediately follows.

B Proof of Theorem 2

We only prove the theorem with $IC_1(q)$, as the proof for $IC_2(q)$ is similar. We denote by C_i some generic positive constants. First we consider $q < r$. We need to show that

$$\frac{1}{d^2}\sum_{k=q+1}^{r} \hat{\lambda}_k > \tau_1(r-q)g(n) \tag{4}$$

holds with probability converging to 1. Let $\mathcal{A} = \{d^{-2}\sum_{k=q+1}^{r}\hat{\lambda}_k \geq d^{-2}\left(C_1 - C_2 n^{-1/2}\right)\}$. Since $d^{-2}\sum_{k=q+1}^{r}\hat{\lambda}_k = d^{-2}\sum_{k=q+1}^{r}(\lambda_k + \hat{\lambda}_k - \lambda_k)$, it follows from Proposition 1 that $P(\mathcal{A}) \to 1$ for appropriately chosen C_1 and C_2. As $g(n) \to 0$, (4) holds on the set $\mathcal{A}$ for all sufficiently large n. For $q > r$, we need to show that $\frac{1}{d^2}\sum_{k=r+1}^{q}\hat{\lambda}_k < \tau_1(q-r)g(n)$ with probability converging to 1. This is true as the sum on the LHS of the above expression is of the order $O_p(1/n)$ and $ng(n) \to \infty$. This completes the proof.