The Auto-Regression and the Moving-Average

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Abstract

In this paper, we aim at writing down explicitly the Gaussian likelihood of T successive observations generated by an ARMA(p, q) process. First, we consider a causal auto-regression of order q and we give a generalized form of the Yule-walker equations. Using these equations, we are able to derive the explicit form of the inverse variance-covariance matrix of T consecutive observations generated by the auto-regression. Then, we consider a moving-average process with the same parameters. The moving-average is a function of present and future values of the same white noise sequence as the auto-regression. For T consecutive observations from the moving-average, we show that the inverse variance-covariance matrix is a conditional variance matrix. Finally, after considering a second causal auto-regression of order p, based on the past and present values of the same white-noise sequence, we define a causal and invertible ARMA(p, q) process. We collect $T > 2\max\{p, q\}$ observations and are interested in their inverse covariance matrix. We show that it is a conditional variance matrix too. For Gaussian series, we propose an algorithm that computes the exact likelihood function of the unknown parameters based on T successive observations from the ARMA(p, q) process.